Dynamics, Topology and Computations

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Abstracts

Arai, Zin (Kyoto University, Japan), *A Hyperbolicity Verification Algorithm and its Application*
In this talk, we propose a rigorous computational method to prove the uniform hyperbolicity of discrete dynamical systems. Applying the method to the real and complex Hénon family, we prove that there are many regions of hyperbolic parameters and the parameter space of the family has very rich and interesting structure. In particular, we prove one of John Hubbard's conjecture about the property of hyperbolic horseshoe loci of the complex Hénon family.

Arioli, Gianni (Politecnico di Milano), *A functional analytic approach to computer assited proofs.*
In this talk we describe a recent approach to computer assisted proofs in the study of differential equations. This method heavily relies on an object oriented programming technique, where the objects are elements of a suitably chosen functional space. The first application of this technique concerns the existence of periodic solutions for the Fermi-Pasta-Ulam Model, see Comm. Math. Phys. 255, 1, (2005) 1-19. The technique has been subsequently applied to the study of the following problems: bifurcation graph for stationary solutions of infinite dimensional dynamical systems, coreographic solutions for the 3 body problem, radial solutions for elliptic and biharmonic equations. We discuss pros and cons of this approach with respect other well known techniques, and future developments.

Barrio, Roberto (University of Zaragoza, SPAIN), *Chaos Indicators and spurious structures*
One of the most important and, why not to say, most popular mathematical theories of the XX century is the Chaos theory. Therefore, an analysis of the regular and chaotic behaviour of a dynamical system has became more and more important. To study the chaoticity of a system in a rigorous manner is not an easy task and moreover, it is not always possible. Therefore, a fast and easy technique that permits us to locate regions where chaos is probable will be a very useful one. The last few years has appeared several of such a techniques (Chaos Indicators) but most of them may give us wrong results or at least some spurious structures when they are used without care. The main objective of this talk is to study some Chaos Indicators and to try to study when such a spurious structures appear and how to avoid some of them.

Among the plethora of Chaos Indicators we have selected the following methods:

- **Variational methods** explore the behaviour of variations vector associated with a given fiducial trajectory. We have studied the behaviour of three variational indicators: MEGNO, FLI and OFLI2.
- **Spectral methods**. Typical spectral methods focus on a single orbit, paying no attention to its neighborhood in the phase space. This point makes them essentially different from variational methods. We are going

to confront variational methods with simple counting of the frequency peaks (Spectral Number) and with the integrated autocorrelation function IACF.

Barutello, Vivina (Università di Milano-Bicocca), *A bisection algorithm for the numerical Mountain Pass*
We propose a new algorithm, based on a bisection method, to determine critical points with non-zero Morse index. This algorithm needs a very low number of steps to obtain good approximations of critical points.

We use it to determine a new numerical solution for the nonplanar 3-body problem; we then give a computer assisted proof for the existence of this solution.

Batko, Bogdan (WSB-NLU), *On the homology of representable sets*
Joint work with Prof. Marian Mrozek.
In this talk we construct the homology of representable sets and investigate its basic properties. Recall that a subset of $\mathbb{R}^d$ is representable (cf. [T. Kaczynski, K. Mischaikow, and M. Mrozek, *Computational Homology*, Appl. Math. Sci. Vol. 157, Springer Verlag, NY 2004]) if it is a finite union of elementary cells $\overset{\circ}{I}_1 \times \overset{\circ}{I}_2 \times \cdots \times \overset{\circ}{I}_d$, where

$$\overset{\circ}{I}_k = \begin{cases} (l, l+1) \\ [l] \end{cases} \quad k \in \{1, 2, \ldots d\}.$$

Benedicks, Michael (Royal Institute of Technology), *Perturbation based and computer aided proofs for the existence of chaotic attractors*
In this talk I will discuss the various methods that exist to prove chaotic behaviour for some simple low-dimensional dynamical systems. For certain systems like the quadratic maps and the Hénon maps the methods to prove expansion and positive Lyapunov exponent are perturbative and based on an induction to select parameters.

In other cases, like the Lorenz map, the methods are based on rigorous computer calculations combined with analysis. I will describe what has been achieved with these two methods and discuss whether they can be combined.

Berz, Martin (Michigan State University), *Rigorous High-Order Computational Methods*
We discuss approaches that allow the rigorous computational treatment of functional dependencies that scale with a high order of discretization size. The methods are based on arithmetic on a data type comprised of an approximate high-order multivariate Taylor expansion of the functional dependencies and a simultaneous rigorous treatment of a bound of the remainder. As such it represents a hybrid between the rigor of formula manipulation and the versatility of conventional numerical techniques.

For discrete systems, the method allows topological arguments about the flow merely by evaluating functional relationships. For continuous systems, the arithmetic is combined with fixed point theorems on function spaces to provide rigorous enclosures of the flow, as shown in detail in a companion talk.

The resulting representations are used to rigorously study various questions arising in topological analysis of dynamics to high precision. In particular we study the questions of containment of isolated points in sets, and generalize to the question of intersection or disjointness of sets. Furthermore, we study the general question of global optimization and range bounding, which can be used to express very general topological problems.

The methods have been implemented in the COSY INFINITY software system, focusing on computational efficiency and resulting in accuracies often approaching that of the underlying floating point system. Various examples will be given including use of the methods to determine very sharp and rigorous enclosures of Poincare maps, unstable and stable manifolds, and homoclinic points.

Capiński, Maciej (Jagiellonian University, Poland), *Transition chains in the planar restricted elliptic three body problem*
In the planar restricted circular three body problem, for the values C of the Jacobi constant smaller but close to the value C_2 associated with the critical point L_2, there exists a family of the Lapunov periodic orbits around the equilibrium point. We will show that when the planar restricted elliptic three body problem is considered as a perturbation of the circular problem, most of the Lapunov orbits persist and are perturbed into a Cantor set of invariant tori. We will also show that there exist transition chains between the tori, which arise from transversal intersections of the corresponding invariant manifolds. These intersections are not restricted to a constant energy manifold and each transition involves a change of energy.

Celletti, Alessandra (Universita' di Roma "Tor Vergata" (Italy)), *Stability of a 3-body problem in Celestial Mechanics*
KAM theory is a powerful tool apt to prove perpetual stability in Hamiltonian systems, which are a perturbation of integrable ones. The smallness requirements for its applicability are well known to be extremely stringent. A long standing problem, in this context, is the application of KAM theory to "physical systems" for "observable" values of the perturbation parameters. In a joint work with L. Chierchia, we consider the Restricted, Circular, Planar, Three-Body Problem (RCPTBP). When the mass ratio of the two primary bodies is small, the RCPTBP is described by a nearly-integrable Hamiltonian system with two degrees of freedom. We consider the motion of the asteroid 12 Victoria, taking into account only the Sun-Jupiter gravitational attraction. For values of mass ratios up to $1/1000$, we prove the existence of two-dimensional KAM tori on a fixed three-dimensional energy level. The proof is based on: 1) an iso-energetic KAM theory; 2) an algorithm for computing iso-energetic, approximate Lindstedt series; 3) a computer-aided application of 1)+2) to the

Sun-Jupiter-Victoria system.

Collins, Pieter (Centrum voor Wiskunde en Informatica), *Computability in Dynamical Systems Theory*
Computational methods have had played a vital role in the development of modern dynamical systems theory. Many early metho ds were concerned with computing approximations to orbits and measures of chaos. Recently, there has been a growing intere st in rigorous computation, in which numerical results can be used to prove properties of the real model.

Which quantities can be computed to arbitrary accuracy? If a quantity is not computable, then what are the best possible c omputable approximations? More fundamentally, how do we even describe arbitrary points, sets and systems, given that these can take uncountably many values?

In this talk, I will discuss Weihrauch's computable analysis theory, which provides a formal methodology for discussing co mputation on points, sets and functions. I will then describe the Ariadne project, in which these ideas are being implemen ted in a general-purpose tool for system analysis. Finally, I will briefly give some applications to dynamical systems, in cluding computation of limit sets, periodic orbits, index theory and topological entropy.

Ćwiszewski, Aleksander (N. Copernicus Univ., Toruń), *Linearization method for homotopy invariants of perturbations of m-accretive operators*
A concept of resolvent differentiability of m-accretive operators shall be introduced. By use of the linearization, we provide a topological index formula for an isolated zero x_0 of a map $-A+F$ where $F: U \to E$ is a differentiable map defined on an open neighborhood U of x_0, A is a m-accretive operator with compact resolvent and admitting a resolvent derivative $\widetilde{A} : D(\widetilde{A}) \to E$ at x_0 such that the C_0 semigroup $\{e^{-t\widetilde{A}}\}_{t\geq 0}$ is norm continuous on $(0, +\infty)$. Moreover, the necessary and sufficient criteria for bifurcation, in terms of resolvent derivative, are stated. The second part of the talk is devoted to the homotopy index of a local semiflow Φ generated by the evolution inclusion $\dot{u} \in -Au+F(u)$ where $A : D(A) \multimap E$ is a m-accretive operator generating compact semigroup and $F : E \to E$ is locally Lipschitz map. It is verified that Φ is admissible in the sense of the Rybakowski homotopy index. Next, a method for computation of the homotopy index of an isolated equilibrium of Φ by means of the resolvent derivative of A at the equilibrium point is provided. Finally, an example of application to partial differential equations with nonlinear boundary conditions will be given.

Derivière, Sara (Universite de Sherbrooke, Canada), *Dynamical System Frameworks for Surface Modeling and Image Recognition, Part II*
We define a new mathematical model, based on the concept of multivalued discrete dynamical system, for computing topological shape descriptors of three dimensional objects. Discrete analogies of a Morse function, its gradient field, and its stable and unstable manifolds are established in order to interpret func-

tions numerically given on finite sets of pixels. We present efficient algorithms detecting critical components of a function f and displaying connections between them by means of a graph, called the *Morse connections graph* whose nodes represent the critical components of f and edges show the existence of connecting trajectories between nodes. This graph structure is extremely well suitable for shape comparison and shape matching and it inherits invariant properties of a given Morse function. This is a joint work with M. Allili, D.Corriveau, T. Kaczynski and A. Trahan. I will present

Part II - experiments

while T. Kaczynski will present

Part I - mathematical models.

Fura, Justyna (Nicolaus Copernicus University, Poland), *Periodic solutions of second order Hamiltonian systems bifurcating from infinity*
We study connected sets of periodic solutions of autonomous second order Hamiltonian systems emanating from infinity. Namely, we consider the following family of autonomous second order Hamiltonian systems

$$\begin{cases} \ddot{u} = -V'(u,\lambda) \\ u(0) = u(2\pi) \\ \dot{u}(0) = \dot{u}(2\pi) \end{cases} \tag{1}$$

where $V \in C^2(\mathbb{R}^n \times \mathbb{R}, \mathbb{R})$ and gradient V' is asymptotically linear, i.e.

$$V'(u,\lambda) = A(\lambda) + o(\| u \|),$$

as $\| u \| \to \infty$ locally uniformly in bounded intervals and $A(\lambda)$ is real symmetric matrix for any $\lambda \in \mathbb{R}$.

As the main tool we apply the degree for $SO(2)$-equivariant gradient operators. It is worth in pointing out that application of classical invariants like the Conley index technique and the Morse theory does not ensure the existence of closed connected sets of critical points of variational problems. Also the Leray-Schauder degree is not applicable in considered situation.

We prove sufficient conditions for the existence of connected sets of 2π-periodic solutions of (1) emanating from infinity. We formulate them in terms of the right hand sight of system (1), i.e. potential V. We also indicate points at which an unbounded closed connected set of critical $SO(2)$-orbits meets infinity and describe the minimal periods of solutions of system (1).

Galias, Zbigniew (AGH, Kraków), *On rigorous studies of chaotic attractors of low dimensional continuous time systems*
In this work we describe interval arithmetic methods for rigorous investigations of chaotic attractors generated by continuous time systems. We will show how

to find a trapping region for the system, compute rigorous estimates for the average return time of the Poincaré map, and find all short periodic orbits for the system. We will also discuss the problem of rigorous integration of the system in case the left hand side of the differential equation is not smooth.

We consider two examples. The first one is the third order smooth system

$$\begin{aligned} \dot{x_1} &= -x_2 - x_3, \\ \dot{x_2} &= x_1 + a x_2, \\ \dot{x_3} &= b + x_3(x_1 - c) \end{aligned}$$

with $a = b = 0.2$, $c = 5.7$ for which the Roessler attractor is observed.

As a second example we consider a simple third order piecewise linear system (Chua's circuit) exhibiting complex trajectories

$$\begin{aligned} C_1\dot{x}_1 &= (x_2 - x_1)/R - g(x_1), \\ C_2\dot{x}_2 &= (x_1 - x_2)/R + x_3, \\ L\dot{x}_3 &= -x_2 - R_0 x_3 \end{aligned}$$

where $g(z) = G_b z + 0.5(G_a - G_b)(|z+1| - |z-1|)$ is a three segment piecewise linear characteristics. The circuit is studied with the following parameter values $C_1 = 1$, $C_2 = 7.65$, $G_a = -3.4429$, $G_b = -2.1849$, $L = 0.06913$, $R = 0.33065$, $R_0 = 0.00036$, for which a chaotic attractor is observed in computer simulations.

Garay, Barnabas (Budapest Univ. of Technology), *Optimization and the Miranda approach in detecting horseshoe–type chaos by computer*
We report on experiences with an adaptive subdivision method supported by interval arithmetic that enables us to prove subset relations of the form $\mathcal{T}(W) \subset U$ and thus to ckeck certain sufficient conditions for chaotic behaviour of dynamical systems in a rigorous way. Our proof of the underlying abstract theorem avoids of referring to any results of applied algebraic topology and relies only on the Brouwer fixed point theorem. The second novelty is that the process of gaining the subset relations to be checked is, to a large extent, also automatized. The promising subset relations come from solving a constrained optimization problem via the penalty function approach. Abstract results and computational methods are demonstrated by reconsidering iterates of the Hénon mapping as well as Hubbard's pendulum equation. This is joint work with Balázs Bánhelyi, Tibor Csendes, and László Hatvani.

Gawrycka, Joanna (N. Copernicus University, Toruñ, Poland), *Solutions of Multiparameter Systems of Elliptic Differential Equations*
The aim of the talk is to study behaviour of weak solutions of the following system of elliptic differential equations:

$$\begin{cases} -\Delta u = \Lambda u + \nabla_u \eta(u, \Lambda) & \text{in } \Omega \\ \quad u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where

1. $\Omega \subset \mathbb{R}^n$ is an open, bounded set with a C^{1-}boundary,

2. $\Lambda \in S(m, \mathbb{R})$, where $S(m, \mathbb{R})$ denotes the set of real, symmetric $(m \times m)$-matrices,

3. $\nabla_u \eta(x, \Lambda) = o(|x|)$ locally uniformly in $\Lambda \in S(m; \mathbb{R})$.

The matrix $\Lambda \in S(m, \mathbb{R})$ is considered as a parameter.

Let us consider the Sobolev space $\mathbb{H} = \mathbb{H}_0^1(\Omega) \oplus \ldots \oplus \mathbb{H}_0^1(\Omega) = \bigoplus_{i=1}^{m} \mathbb{H}_0^1(\Omega)$ with the inner product defined as follows:

$$\langle v, w \rangle_{\mathbb{H}} = \sum_{i=1}^{m} \langle v_i, w_i \rangle_{\mathbb{H}_0^1(\Omega)} = \sum_{i=1}^{m} \int_{\Omega} \nabla v_i \nabla w_i,$$

for $v = (v_1, \ldots v_m), w = (w_1, \ldots, w_m) \in \mathbb{H}$.

Our purpose is to study the set of bifurcation points, branching points and global bifurcation points of the solutions of system (2). With the system (2) we associate a C^2-functional $\Phi : \mathbb{H} \times S(m; \mathbb{R}) \to \mathbb{R}$ whose critical points (with respect to u) are in one-to-one correspondence to the solutions of the system (2). We study critical points of this functional.

In the talk we consider two cases: symmetric and non-symmetric Ω.

In non-symmetric case we reduce a local problem of studies of the set $(\nabla_u \Phi)^{-1}(0)$ in a neighborhood of the point $(0, \Lambda) \in \mathbb{H} \times S(m, \mathbb{R})$ to a finite-dimensional one, and then we describe the set of bifurcation points of the solutions of system (2). Finally, applying the Leray-Schauder degree, we describe the set of global bifurcation points of the solutions of system (2).

The main tool we use in symmetric case is the theory of the degree for SO(2)-equivariant gradient maps. We treat the space $\mathbb{R}^n$ as an orthogonal representation of the group $SO(2)$ and assume that $\Omega \subset \mathbb{R}^n$ is $SO(2)$-invariant. Hence $(\mathbb{H}, \langle \cdot, \cdot \rangle_{\mathbb{H}})$ is an orthogonal representation of the group $SO(2)$ with $SO(2)$-action given by the formula $(g \cdot v)(x) = v(gx)$ for any $g \in SO(2)$ and $v \in \mathbb{H}$. We derive formula for bifurcation index in the terms of the degree for $SO(2)$-equivariant gradient operators and using Rabinowitz global bifurcation theory we describe the set of global bifurcation points of the solutions of system (2).

The set of global bifurcation points obtained in symmetric case is bigger than the set obtained in non-symmetric one. It shows the advantages of using the degree for $SO(2)$-equivariant gradient maps.

Ghrist, Robert (University of Illinois, Urbana), *Homological Methods for Sensor Networks*

As sensor engineering and manufacturing evolve to produce smaller devices, we will face the problem of dealing with large collections of local networked devices. What types of global problems can be solved by a swarm of local sensors? Topologists solved a similar problem nearly a century ago, and in so doing invented some very sophisticated algebraic tools. This talk will demonstrate the surprising effectiveness of algebraic topology as a toolbox for working with sensor networks having neither localization capabilities nor probabilistic assumptions.

Gołębiewska, Anna (UMK Toruñ, Poland), *Degree Theory for Equivariant Strongly Indefinite Operators*
We study the following system of elliptic equations:

$$(E)\begin{cases}\triangle u_1(x) = \nabla_{u_1} F(u_1(x), u_2(x), \lambda) \text{ for } x \in \Omega,\\ -\triangle u_2(x) = \nabla_{u_2} F(u_1(x), u_2(x), \lambda) \text{ for } x \in \Omega,\\ u_{|\partial\Omega} = (u_{1|\partial\Omega}, u_{2|\partial\Omega}) = 0 \in \mathbb{R}^2,\end{cases}$$

where $\nabla_u F(u,\lambda) = \lambda u + \nabla_u \eta(u,\lambda)$, and for all $\lambda \in \mathbb{R}:\ \nabla_u \eta(0,\lambda) = 0,\ \nabla_u^2 \eta(0,\lambda) = 0$ and there are constants $C > 0$ and $1 \leq p_\lambda < (N+2)(N-2)^{-1}$ such that $|\nabla_u F(u,\lambda)| \leq C(1+|u|)^{p_\lambda}$.
It is known that the weak solutions of the system (E) are in one-to-one correspondence with the critical points of a C^2- functional Φ defined by

$$\Phi(u,\lambda) = \frac{1}{2}\int_\Omega (-|\nabla u_1|^2 + |\nabla u_2|^2)dx - \int_\Omega F(u,\lambda)dx.$$

Since $\nabla_u \Phi$ is a strongly indefinite operator, the degree for G-equivariant operators cannot be applied. Using this degree we define the degree of strongly indefinite G-equivariant operators in the form of the compact perturbation of Fredholm operator of index 0.

We formulate an abstract bifurcation theorem of the Rabinowitz type. Applying this theorem to the functional Φ defined for the system (E) we obtain conditions for existence of bifurcation points of this functional.

Hampton, Marshall (University of Minnesota Duluth), *Orbits and dynamics of the four vortex-problem*
There is a wealth of structure in the dynamics of point vortices. This system was introduced by in the late 19th century to model vortex filaments in an ideal fluid. Since then, the dynamics of three vortices has become well understood, and the model has been generalized in many ways (to surfaces other than the plane, for example). The planar model has an unusal Hamiltonian structure in the context of point particle dynamics which has not been fully exploited. This talk will focus on the four-vortex problem. Depending on the vortex strengths, this can be an integrable or chaotic system, but most questions about the dynamics remain unanswered. Besides giving some historical context for this problem, some new results, which are joint with Richard Moeckel, will be presented on the existence and properties of self-similar orbits.

Hell, Juliette (Freie Universität Berlin), *Conley-Index of infinity*
The Conley index is a topological tool used, among other things, to analyse the structure of connecting orbits. Therefore it seems appropriate to use the

Conley index of infinity to analyse the relationship between the set of bounded solutions and of solutions going to infinity in a dynamical system. The aim is to be able to understand the interplay between the set of eternal solutions (bounded solutions defined for all time) and the set of solutions which become unbounded in finite or infinite time.

Unfortunately any neighborhood of infinity is unbounded, so that we have to "compactify" the space for the Conley index of infinity to make sense.However, even for finite dimensional systems, the naive idea of defining the Conley index as usual after the compactification, is not sufficient to treat all interesting cases: the isolation of infinity is often a problem. Therefore a construction using the Poincare-Lefschetz duality will be introduced to deal with the lack of isolation. To illustrate which cases may be analysed with this tool, quadratic vector fields in the plane will be discussed as an example.This construction can be also used for other degenerate situations.

Furthermore the limitations of this tool will be presented in the case of infinite dimensional dynamical systems. This is work in progress.

Junge, Oliver (Munich University of Technology), *Rigorous numerics for infinite dimensional maps*
We present a computational technique that allows to draw rigorous conclusions about the global dynamical behaviour of infinite dimensional maps.

Based on a finite dimensional Galerkin approximation of the system and corresponding bounds on the truncation error, the technique employs set oriented numerical methods for the computation of coverings of invariant sets of the multivalued Galerkin system. Using the Conley index theory, rigorous statements about the dynamical behaviour within these coverings are obtained. By verifying certain conditions on the truncated modes it is possible to lift the index information to the full system. Depending on the complexity of the dynamics, it is possible to localize the corresponding invariant sets up to machine precision.

Together with the theory we present the algorithms behind this approach, as well as a numerical example. This is joint work with Sarah Day (Cornell University) and Konstantin Mischaikow (Georgia Institute of Technology).

Kaczynski, Tomasz (Université de Sherbrooke), *Multivalued Discrete Dynamical System Framework for Surface Modelling, Part I*
We define a new mathematical model, based on the concept of multivaued discrete dynamical system, for computing topological shape descriptors of three dimensional objects. Discrete analogies of a Morse function, its gradient field, and its stable and unstable manifolds are established in order to interpret functions numerically given on finite sets of pixels. We present efficient algorithms detecting critical components of a function f and displaying connections between them by means of a graph, called the *Morse connections graph* whose nodes represent the critical components of f and edges show the existence of connecting trajectories between nodes. This graph structure is extremely well suitable for

shape comparison and shape matching and it inherits invariant properties of a given Morse function. This is a joint work with M. Allili, D. Corriveau, S. Derivière and A. Trahan. I will present

Part I - mathematical model

while S. Derivière will present

Part II - experiments.

Kalies, William (Florida Atlantic University), *A Computational Approach to Conley's Decomposition Theorem*
By discretizing the phase space, a combinatorial representation of a dynamical system can be obtained from an appropriate multivalued map. This technique, combined with information from the Conley index, has been used to establish rigorously the existence of various types of dynamics in both finite and infinite-dimensional systems. In this talk we describe algorithms for approximating the chain recurrent set, for computing Morse decompositions, and for computing weak discrete Lyapunov functions which approximate continuous Lyapunov functions for the underlying system.

Kiss, Gábor (University of Szeged), *Stability conditions for linear autonomous functional differential equations*
This paper deals with stability conditions and stability charts of functional differential equations with two parameters. The investigated equations are given with generally and symmetrically distributed delays. Our results are generalizations and completion of outcomes found in the paper of Bernard, S., Bélair, J., Mackey, M. C. [*Discrete and continuous dynamical systems* **2** (2001), 233-256]. Statements are formulated by the expected value of delays. It has importace in applications if the delays aren't known exactly, but they are given with their mean. The argumentation in the proofs of present paper are based on the characteristic function of the given equations. We exploit the fact that the roots of the characteristic function change continously if we change a parameter continuously in our equation.

Kokubu, Hiroshi (Kyoto University, Japan), *Conley-Morse chain complexes and chain maps based on spectral sequences*
Given a Morse decomposition of an isolated invariant set, an approach using spectral sequences based on its totally ordered coarsenings is presented for constructing a Conley-Morse chain complex (a connection matrix) and a Conley-Morse chain map (a transition matrix). By this approach, it can be shown the existence of a Conley-Morse chain map that covers the continuation isomorphisms of any intervals of a totally ordered coarsening that continues across a parameter interval, which may be considered as a generalization of the topological transition matrix of McCord-Mischaikow (in the context of totally ordered coarsenings).

LaValle, Steven M. (University of Illinois), *Minimum Wheel-Rotation Paths for Differential-Drive Mobile Robots*
We have characterized the shortest paths in the plane for differential-drive mobile robots in the spirit of Dubins curves and Reeds-Shepp curves for car-like robots. A differential drive is a mechanism consisting of two independently controlled wheels attached to a common axle. This is the most common drive mechanism in mobile robotics. For this model, a well-defined notion of shortest is obtained by optimizing the total amount of wheel rotation. We establish that shortest paths indeed exist for this problem and that the shortest paths can be expressed by piecewise-constant motor controls. From this, the shortest paths can be organized into 52 classes, in which each is generated by a unique control string. The results follow from Pontryagin's Maximum Principle, the Sussmann-Tang lemma, and further geometric arguments that are particular to the differential drive.

This is joint work with Hamidreza Chitsaz (UIUC), Devin Balkcom (Dartmouth), and Matt Mason (Carnegie-Mellon Univ.).

Lessard, Jean-Philippe (Georgia Tech), *Validated Continuation for Equilibria of PDE's*
One of the most efficient methods for determining the equilibria of a continuous parameterized family of differential equations is to use predictor-corrector continuation techniques. In the case of partial differential equations this procedure must be applied to some finite dimensional approximation which of course raises the question of the validity of the output. We introduce a new technique that combines the information obtained from the predictor-corrector steps with ideas from rigorous computations and verifies that the numerically produced equilibrium for the finite dimensional system can be used to explicitly define a set which contains a unique equilibrium for the infinite dimensional partial differential equation. Use the Swift-Hohenberg equation as a model we demonstrate that the cost of this new validated continuation is less than twice the cost of the standard continuation method alone.

de la Llave, Rafael (U. Texas at Austin), *The parameterization method for invariant manifolds. Examples of breakdown of normal hyperbolicity*
This is joint work with Alex. Haro.

We present the parameterization method to compute normally hyperbolic manifolds.

We use it to study some examples of manifolds close to the breakdown of normal hyperbolicity. We empirically find some universal behavior and formulate some conjectures about the mechanisms that lead to loss of normal hyperbolicity.

Lust, Alexander (University of Bielefeld, Bielefeld, Germany), *A hybrid method for computing Lyapunov exponents*
In this talk we present a new numerical method for computing some or all Lyapunov exponents of a discrete dynamical system. It is called hybrid because it

combines the classical QR-method (see [4], [5]) with more recent methods that use spatial integration with respect to an invariant ergodic measure.

More specifically, we approximate the j-th Lyapunov exponent λ_j of a map $g : M \mapsto M$ by the sequence $\{a_n^j\}$ of integrals

$$a_n^j = \frac{1}{n} \int \ln R_{jj}(Dg^n(x)) d\mu$$

where $R_{jj}(Dg^n(x))$ is the j-th diagonal element of the R-matrix in the unique QR decomposition of the Jacobian $Dg^n(x)$ and μ is an invariant ergodic measure for g. The method extends the approach of Aston and Dellnitz (see [1], [2]) who used the sequence $\{a_n\}$ of integrals

$$a_n = \frac{1}{n} \int \ln \|Dg^n(x)\| d\mu$$

in order to approximate the dominant Lyapunov exponent λ_1. As in their approach we use the package GAIO (see [3]) in order to compute an approximate invariant measure. Combining this with the QR decomposition leads to a robust method for computing all Lyapunov exponents which, in fact, yield better approximations even for the dominant exponent. This will be illustrated by several numerical examples.

We also prove error expansions for the approximate exponents above. These are of the form

$$a_n^j = \lambda_j + \frac{c_j}{n} + o\left(\frac{1}{n}\right)$$

and

$$a_n^j = \lambda_j + \frac{c_j}{n} + o\left(\frac{e^{-\delta n}}{n}\right),$$

where the constants c_j are determined by integrating certain exterior products of Oseledets vectors with respect to the invariant measure and $\delta > 0$. For the proof of these expansions we assume that there is a gap in the Lyapunov spectrum and that corresponding Oseledets spaces are well separated.

References

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Luzzatto, Stefano (Imperial College London), *A computer-assisted proof in one-dimensional dynamics*
I will give a brief survey of some recent qualitative results in one-dimensional dynamics, focussing on the problem of the occurrence of stochastic (chaotic) dynamics in families of maps. I will then discuss the extent to which a rigorous quantitative understanding is possible. i.e. can we actually determine whether the dynamics is chaotic for a particular parameter value ?

In this direction will present some recent results, joint with H. Takahasi, on rigorous estimates for the probability of chaotic dynamics in the quadratic family. I will also discuss theoretical and computational issues involved in extending these results to more general families of dynamical systems. This is ongoing work in progress with S. Day, H. Kokubu, and K. Mischaikow.

Maier-Paape, Stanislaus (RWTH Aachen), *Rigorous numerics to verify heteroclinic connections*
Abstract:
We consider a gradient system $\dot{x} = -\nabla g(x)$, $x \in \mathbb{R}^n$ with two hyperbolic equilibria $x_R, x_A \in \mathbb{R}^n$ with Morse indices Σ^{k+1} and Σ^k $(0 \leq k \leq n-1)$, respectively. From purely computational investigations we have some numerical "evidence" that there could be a heteroclinic connection between x_R (repeller) and x_A (attraktor), which is transverse (i.e. the stable $(W^s(x_A))$ and unstable $(W^u(x_R))$ manifolds intersect transversely.
Our goal is the construction of a rigorous numerical method (combined analysis and numerical verification) that indeed proves the existence of a heteroclinic connection between x_R and x_A.
Our methods works in principle for $n \in \mathbb{N}$, but due to current gaps in the proof of **Phase 3** for $n > 5$, at this point we can only announce the result for $n \leq 4$.

Makino, Kyoko (Michigan State University), *Taylor Model-based Verified Integration of ODEs*
We discuss the foundations of Taylor Model-based verified integrators. Compared to other verified tools, these methods originally developed in the field of beam physics tools allow the representation of the flow of extended domains with very little overestimation. In particular we discuss various preconditioning methods, including the method of curvilinear coordinates and various blunting approaches. We illustrate the performance with a comprehensive collection of examples.

Maksymenko, Sergiy (NAS of Ukraine, Kiev, Ukraine), *Smooth shifts along orbits of vector fields*

Let $f : R^n \to R^1$ be the germ of smooth (C^∞) function such that $f(0) = 0$ and let $\phi : R^1 \to R^1$ be the germ of preserving orientation diffeomorphisms at 0, i.e. $\phi(0) = 0$ and $\phi'(0) > 0$.

Problem. Find a germ of a diffeomorphism $h : R^n \to R^n$ such that $h(0) = 0$ and

$$\phi(f(x)) = f(h(x)). \tag{3}$$

This equation arises from singularities theory.

Suppose that f satisfies the following condition (*): there are smooth functions $\alpha_1, \ldots, \alpha_n$ such that

$$(*) \qquad f(x) = \alpha_1 f'_{x_1} + \cdots + \alpha_n f'_{x_n}.$$

This condition is rather general, and holds for many singularities, e.g. if $f(x) = \pm x^2 \pm y^2$, then $f = \frac{x}{2} f'_x + \frac{y}{2} f'_y$.

Theorem. *If f satisfies (*), then for arbitrary ϕ there exists a solution h_ϕ of (3). Moreover, the correspondence $\phi \mapsto h_\phi$ is a homomorphism of the corresponding groups of germs of diffeomorphisms $Diff(R^1) \to Diff(R^n)$.*

If we define a vector field $F = (\alpha_1, \ldots, \alpha_n)$, then (*) can be rewritten as follows: $f = df(F)$. Let $\Phi : R^n \times (-\varepsilon, \varepsilon) \to R^n$ be a local flow of F. I show that the *exact* solution of (3) can be represented as a *smooth shift along orbits of vector field F*. By the Hadamard lemma each ϕ can be represented in the form $\phi(t) = t\bar{\phi}(t)$, where $\bar{\phi}$ is smooth that $\bar{\phi}(0) = \phi'(0) > 0$. Let $\sigma_\phi(t) = \ln \bar{\phi}(t)$. It turs out that

$$h_\phi(x) = \Phi(x, \sigma_\phi(f(x)) = \Phi(x, \ln \bar{\phi}(f(x)).$$

is a solution of (3).

I shall also discuss the usefulness of such smooth non-constant shifts along orbits of vector fields to some other problems.

Marzantowicz, Wacław (UAM, Poznań, POLAND), *A symmetry implies chaos for a sphere mapping*

The well-known example, given by Shub, shows that for any $|d| \geq 2$ there is a self-map of the sphere S^n, $n \geq 2$, of degree d for which the set of non-wandering points consists of two points. It is a natural to ask which additional assumptions guarantee the infinite number of periodic points of such a map. We show that if a continuous map $f : S^n \to S^n$ commutes with a free homeomorphism $g : S^n \to S^n$ of a finite order, then f has infinitely many minimal periods, and consequently infinitely many periodic points. In other words the assumption of the symmetry of f originates a kind of chaos. We also give an estimate of the number of periodic points of m-th iteration of f.

Mrozek, Marian (Jagiellonian Univ. and WSB-NLU, Poland), *Coreduction Homology Algorithm*

We present a new algorithm for computing homology of cubical sets. The algorithm is based on homology theory of representable sets and elementary coreductions, a concept dual to elementary reductions.

Muchewicz, Krzysztof (N. Copernicus University, Toruñ, Poland), *Solutions of elliptic equations with Neumann boundary conditions*
Let us consider the following equation

$$\begin{cases} -\Delta u &= f(u) \quad in\ \Omega, \\ \dfrac{\partial u}{\partial \nu} &= 0 \qquad on\ \partial\Omega, \end{cases} \tag{4}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded subset with boundary of class C^{1-} and $f \in C^1(\mathbb{R}, \mathbb{R})$. Assume that

- $\# f^{-1}(0) < \infty$,
- $\lim_{|t| \to \infty} f(t)$ exists.

The aim of my talk is to formulate sufficient conditions for the existence of week nonconstant solutions of equation (4).

Nishiura, Yasumasa (RIES, Hokkaido University), *Application of the computational homology to complex morphology*
In this talk, we discuss two things: first we apply the computational homology approach to reduce the morphological complexity arising in diblock copolymer melts to simple graphs that still describe important properties of the system. We demonstrate that the betti numbers can be used to distinguish between morphologies quantitatively. Our topological characterization points to the transient perforated lamellar state in the lamellar- hexagons transition and the t-1 law of the Betti number in the late stage of phase-ordering process. Second we use the same approach to understand the spatio-temporal chaos of spotty patterns for reaction diffusion systems. We show that topological quantities refelct the dynamics of splitting and destruction of spots and give us different insight from conventional methods. We also compare this result with the time-development of thermodynamics quantities such as entropy production rate associated with the system. This is a joint work with Dr.Takashi Teramoto.

Nowicki, Tomasz (IBM), *The cut-off phenomenon, on the attractors of distributions on graphs*
We consider the question of existence of a unique invariant probability distribution which satisfies some evolutionary property. The problem arises from the random graph theory but to answer it we treat it as a dynamical system in the functional space, where we look for a global attractor. We consider the following bifurcation problem: Given a probability measure μ, which corresponds to the weight distribution of a link of a random graph we form a positive linear operator Φ (convolution) on distribution functions and then we analyze a family of its exponents with a parameter λ which corresponds to connectivity of a sparse random graph. We prove that for every measure μ (*i.e.*, convolution Φ) and every $\lambda < e$ there exists a unique globally attracting fixed point of the operator, which yields the existence and uniqueness of the limit probability distribution

on the random graph. This estimate was established earlier for deterministic weight distributions (Dirac measures μ) and is known as e-cutoff phenomena, as for such distributions and $\lambda > e$ there is no fixed point attractor. We thus establish this phenomenon in a much more general sense.

Joint work with G.Świrszcz (IBM) and D. Gamarnik (MIT)

Oprocha, Piotr (AGH, Kraków, Poland), *Specification property and dense distributional chaos*
The notion of distributional chaos was introduced by Schweizer and Smítal in [Trans. Amer. Math. Soc., 344 (1994) 737] for continuous maps of the interval. Further, this notion was generalized to three versions d_1C–d_3C for maps on compact metric spaces (see e.g. [Chaos Solitons Fractals, 23 (2005) 1581]). Main result of [*J. Math. Anal. Appl.*, 241 (2000) 181] is that a weakened version of the specification property implies existence of two points scrambled set which exhibit d_2C version of distributional chaos. We show that much more complicated behavior is present in this case. Strictly speaking, there exists uncountable and dense scrambled set consisting of recurrent but not almost periodic points which exhibit d_1C version of distributional chaos.

Pilarczyk, Paweł (Jagiellonian U & Georgia Tech), *Cubical Index Pairs and the Excision Property*
In this talk we introduce a new approach to the algorithmic computation of the Conley index for continuous maps based on combinatorial cubical multivalued maps. The technique is inspired by the theory of the Conley index over a base and involves a different perception of the map on the index pair. The main advantage of our construction is that our cubical sets always have the excision property, which may not always be true in the genral case, as we show in some examples.

The definition of the Conley index is based upon the notion of an index pair. Introducing a cubical grid in $\mathbb{R}^n$ and enclosing a continuous map in a combinatorial cubical multivalued map allows one to compute index pairs automatically. Cubical homology can be further used to effectively compute the homological version of the index. In this way, the Conley index can be used in computer-assisted analysis of qualitative behavior of dynamical systems.

Unfortunately, sometimes the combinatorial objects obtained in the algorithmic construction of an index pair are not suitable for the computation of the homological Conley index. Due to the overestimates in the combinatorial map, the inclusion map that appears in the definition of the index map is not an excision in some cases, and thus the index map is not properly defined.

In this talk we introduce an alternative method of treating the constructed index pair and index map, which allows for using them to compute the homological Conley index, even if the inclusion in question is not an excision.

This is joint work with Kinga Stolot.

Pochinka, Olga (Russia), *On classification of Morse-Smale diffeomorphisms on 3-manifolds*

A diffeomorphism of a compact manifold is called Morse-Smale if its nonwandering set consists of finitely many hyperbolic periodic orbits whose invariant (stable and unstable) manifolds are pairwise transverse. The dynamics of a Morse-Smale diffeomorphism is so simple that it looks easy to classify Morse-Smale diffeomorphisms up to topological conjugacy. However, even on compact surface, this classification has been a long times by hard problem leading to many publications, and the classification has been done at the same times than the classification of a much larger family of diffeomorphisms, including horseshoe and hence chaotic dynamics. The classification of Morse-Smale diffeomorphisms of compact 3-manifolds presents many extra difficulties, for example, the existence of a wild separatrices of saddle points and heteroclinic orbits.

This is joint work with Christian Bonatti and Viatcheslav Grines. Research partially supported by RFBR 05-01-00501, grant of the President of RF supporting leading scientific school 9686.2006.1 (Russia).

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Rybicki, Sławomir (N. Copernicus University, Toruñ, Poland), *Equivariant gradient maps*

Let $\mathbb{V}$ be a real, finite-dimensional, orthogonal G-representation, where G is a compact Lie group. Let $C^1_G(\mathbb{V},\mathbb{R})$ denote the set of G-invariant C^1-maps. If $\phi \in C^1_G(\mathbb{V},\mathbb{R})$ then the gradient $\nabla\phi : \mathbb{V} \to \mathbb{V}$ is a G-equivariant C^0-map.

Denote by $[\nabla\phi : (D(\mathbb{V}), S(\mathbb{V})) \to (\mathbb{V}, \mathbb{V} \setminus \{0\})]^{\nabla}_G$ the set of G-equivariant gradient homotopy classes of G-equivariant gradient C^0-maps, which are nonzero on the sphere $S(\mathbb{V})$.

The aim of my talk is to classify $[\nabla\phi : (D(\mathbb{V}), S(\mathbb{V})) \to (\mathbb{V}, \mathbb{V} \setminus \{0\})]^{\nabla}_G$.

Sakajo, Takashi (Hokkaido Univ.), *Integrable four-vortex motion on sphere with zero moment of vorticity*

We consider the motion of four interacting vortex points on a sphere with unit radius, which defines a Hamiltonian dynamical system. When the moment of vorticity vector, which is a conserved quantity, is zero at the initial moment, the motion is integrable. In this talk, we will give you a topological description of the level curves of the Hamiltonian by using a reduction method to a three-vortex problem. We also discuss if the self-similar collapse of the vortex points is possible.

Simó, Carles (Universitat de Barcelona), *How large are the stability regions around triangular points in the 3D RTBP*

The stability properties of the L_4, L_5 points in the 2D RTBP are well known. For the 3D case it seems that there is no way to prevent diffusion. But one can define a "practical stability", i.e., for very large periods of time. In a similar way one can define "boundaries" of the practical stability region, guided by geometric considerations. A standing problem is the nature of these boundaries. They seem to be related to codimension 1 manifolds in the 6D phase space. A long term project has been started, to identify and compute these manifolds for the mass parameter μ in the range $(0, 0.04]$. A progress report shall be presented, showing results of numerical explorations, the topology of the boundaries and the dynamical objects which sit at these boundaries. Checks on the munerical methods shall be also presented.

Stoffer, Daniel (ETH Zurich), *Delay equations with rapidly oscillating stable periodic solutions*
Delay equations of the form

$$\dot{x} = \mu(-x + f(x(t-1)))$$

are considered where f models either positive or negative feedback.

It is known that for monotone functions f there do not exist rapidly oscillating stable periodic solutions.

In a first part piecewise constant non–monotone feedback functions f are investigated. For piecewise constant f the infinite dimensional system is reduced to a finite dimensional system. It is proved analytically that there exist delay equations admitting rapidly oscillating stable periodic solutions. Previous results were obtained with the aid of computers, but only for one specific piecewise constant feedback function. The present proofs work for whole classes of piecewise constant feedback functions. In the case of negative feedback functions, given an odd integer n, we give sufficient conditions on the two parameters describing the shape of f and on the stiffnes parameter μ such that there exist stable periodic solutions with n zeros per unit time interval. The conditions are believed to be sharp.

In a second part we report on numerical experiments. No proofs are given. We first present some conjectures for piecewise constant nonlinearities. Then we deal with the following smooth feedback functions f:

$$f(x) = ax\exp(-x), \qquad x > 0, 1 < a < 16.999...$$

for nonsymmetric negative feedback and

$$f(x) = a\sin(x), \qquad 1 < |a| < \pi$$

for symmetric positive and negative feedback. There seems to exist a strong relationship between (rapidly oscillating) stable periodic solutions of the delay equation and periodic solutions of the recursion formula $x_{n+1} = f(x_n)$. A number of bifurcation phenomena are presented.

Tabor, Jacek (Jagiellonian University), *On fuzzy differential equations*
We introduce and investigate a new definition of a *fuzzy differential equation*. Contrary to the previously known it generalizes the classical differential equations (and inclusions).

Our main idea is based on the introduction of a special "tangent cone" (an equivalent to the tangent space) for the space of fuzzy sets on $\mathbb{R}^n$.

We also discuss some properties of such equations and some existence results.

Terracini, Susanna (Universita' di Milano Bicocca), *On the variational approach to the n-body problem*
This talk will focus on the recent results obtained by the author in works in collaboration with Davide Ferrario and Vivina Barutello, dealing on the periodic n-body problem, from the perspective of the calculus of variations and minimax theory. These researches were aimed at developing a systematic variational approach to the equivariant periodic n-body problem in the two and three-dimensional space. The purpose of the talk is to expose the main problems and achievements of this approach.

Tucker, Warwick (Uppsala University), *Reconstructing metabolic networks using interval analysis*
Recently, there has been growing interest in the modelling and simulation of bio logical systems. Such systems are often modelled in terms of coupled ordinary di fferential equations that involve parameters whose (often unknown) values corres pond to certain fundamental properties of the system. For example, in metabolic modelling, concentrations of metabolites can be described by such equations, whe re parameters correspond to the kinetic rates of the underlying chemical reactio ns. Within this framework, the increasing availability of time series data opens up the attractive possibility of reconstructing approximate parameter values, t hus enabling the *in silico* exploration of the behaviour of non-trivial dyn amical systems. The parameter reconstruction problem, however, is very challengi ng – a fact that has resulted in a plethora of heuristics methods designed to f it parameters to the given data.

In this talk we present a completely deterministic method for parameter reconstr uction based on interval analysis. We illustrate its utility by applying it to r econstruct metabolic networks. Our method not only estimates the parameters very precisely, it also determines the appropriate network topologies. A major stren gth of the proposed method is that it proves that large portions of parameter sp ace can be disregarded, thereby avoiding spurious solutions.

Wanner, Thomas (George Mason University), *On the Accuracy of Homology Computations for Nodal Domains*
Many partial differential equation models arising in applications generate complex patterns evolving with time which are hard to quantify due to the lack of any underlying regular structure. Such models often include some element of stochasticity which leads to variations in the detail structure of the patterns and forces one to concentrate on rougher common geometric features. From

a mathematical point of view, computational algebraic topology suggests itself as a natural quantification tool and has been used in a variety of settings. In many of these instances, one is interested in the geometry of a nodal domain of a function. The nodal domain is usually approximated using an underlying discretization of the considered partial differential equation — which immediately raises the question of the accuracy of the resulting homology computation. In this talk, I will present a probabilistic approach which gives insight into the suitability of this method in the context of random Fourier series. We will obtain explicit probability estimates, which in turn yield a-priori bounds for the suitability of certain grid sizes. Our results apply to one and two space dimensions. In addition, we address the special case of the Cahn-Hilliard models and show how the grid size has to be chosen as a function of the small model parameter in order to yield reliable results.

Wilczak, Daniel (Jagiellonian University), *A geometric method for some bifurcation problems*
A geometric method for proving some type of bifurcation will be presented.

The method originates from [P. Zgliczy'nski, *Fixed point index for iterations, topological horseshoe and chaos*, Topol. Meth. Nonl. Anal. 8 (1) (1996) 169-177, M. Gidea and P. Zgliczy nski, *Covering relations for multidimensional dynamical systems*, Journal of Differential Equations, 202/1 (2004), 33-58] where a geometric method for proving the existence of periodic orbits and chaotic dynamic for maps in $\mathbb{R}^n$ is presented. The main idea of such a method is to verify some kind of topological transversality called covering relations, among some subsets of the phase space which are called h-sets. The main theorem says that if there exists a loop of covering relations $N_0 \overset{f}{\Longrightarrow} N_1 \overset{f}{\Longrightarrow} \cdots \overset{f}{\Longrightarrow} N_n = N_0$, where N_i, $i = 0, \ldots, n$ are mutually disjoint h-sets then there is a periodic point of f whose image belongs in turn to N_i.

The main idea of the presented method is to define covering relation from an h-set in the parameter space into the phase space of the system under consideration and next use the sequence of covering relations in the phase space. If such a sequence of covering relations is carefully chosen then we can deduce that for a certain parameter value some kind of bifurcation occurs, including codimension one or higher homoclinic or heteroclinic bifurcations.

We will present also the applications of such a method to the Michelson system

$$\begin{array}{rcl} \dot{x} &=& y \\ \dot{y} &=& z \\ \dot{z} &=& c^2 - y - \frac{1}{2}x^2 \end{array} \qquad (5)$$

for the range of parameter values $C = [0.8285, 0.861]$.

Theorem.

1. For each parameter value $c \in C$ the Michelson system (5) is Σ_4 chaotic, i.e. a suitable Poincaré map is semiconjugated to the full shift on four symbols.

2. There exists a countable infinite set of parameter values $c \in C$ for which the system (5) possesses a heteroclinic orbit connecting $(-\sqrt{2}c, 0, 0)$ with $(\sqrt{2}c, 0, 0)$ along one dimensional unstable-stable manifolds.

3. For all parameter values $c \in C$ there exist infinitely many heteroclinic solutions connecting $(\sqrt{2}c, 0, 0)$ with $(-\sqrt{2}c, 0, 0)$.

4. There exists a countable infinite set of parameter values $C_h \subset C$ for which the Michelson system (5) possesses a pair of homoclinic orbits to the equilibrium points. Moreover, the set of accumulation points of C_h contains a Cantor set.

Wójcik, Klaudiusz (Jagiellonian University), *Isolating blocks in dimension 3.* We prove that if ϕ is a flow in $\mathbb{R}^3$ and the origin $S = \{0\}$ is an isolated invariant set then there is an acyclic isolating block B for S.

Wójcik, Wojciech (Vrije Universiteit Amsterdam, NL), *Floer Homology for Braids on the Two-Disc*
We are interested in 1-periodic solutions/periodic points of Hamiltonian systems on the two-disc. Such solutions can be viewed as braids. For various types of braids we can define an invariant via Floer homology. The latter can be computed using the standard Conley index. Applying this tool allows us to obtain various types of forcing results for periodic solutions.

Zalewski, Mikołaj (Jagiellonian University), *Periodic solution of a delay differential equation*
I prove the existence of a periodic orbit in the delay differential equation $x'(t) = K\sin(x(t-1))$. I use the Brower degree to show a solution for each Galerkin projection and from the compactness obtain a solution of the whole system. To compute the degree I use a homotopy for which I need to show that it doesn't have a zero on the boundry. Proving appropriate inequalities is the hardest part and is computer assisted.

Żelawski, Marcin (Jagiellonian University, Poland), *Blood Vessel Architecture Analysis Based on Homology Algorithms*
Homology theory can be successfully used in computer vision and image processing.

The method of analysing the architecture of microcirculation will be presented. This method is based on cubical homology algorithms which can estimate the number of vessel branches and their locations. This approach and counting fractal number were applied to endoscopic pictures of colon mucosa in order to detect some kinds of pathologies (colitis, polyps, vessel malformation, cancer).

The proposed idea can be easily extended to three and higher-dimensional images, for example to analyse of the structure of collagen threads.

Żelazna, Natalia (Jagiellonian University, Poland), *A homology algorithm based on acyclic subspace*
In this talk we consider a new algorithm of computing homology groups of a cubical set, based on computing its maximal acyclic subset. Given a cubical set X and its acyclic subset A, the homology of X and relative homology of a pair (X,A) are related as follows:

$$H_n(X) \cong \begin{cases} H_n(X,A) & \text{for } n \geq 1 \\ \mathbb{Z} \oplus H_n(X,A) & \text{for } n = 0 \end{cases}.$$

We present the method of computing the maximal acyclic subset for cubical sets and compare results obtained via new method of computing homology with some other algorithms.

Zgliczynski, Piotr (Jagiellonian University, Krakow, Poland), *Rigorous numerics for dissipative PDEs*
We describe an algorithm for rigorous computation of evolution of dissipative PDEs with periodic boundary conditions. The algorithm have been implemented for Kuramoto-Sivashinsky PDE with odd and periodic boundary conditions and was used to prove the existence of multiple periodic orbits, both stable and unstable ones.

Main idea of the algorithm: we use Fourier series, the phase space is split into main modes (finite dimension) and the tail (inifinite dimension). The evolution of main modes is governed by differential inclusion obtained from Galerkin projection plus the influence of the tail, the evolution of the tail is governed by linear differential inequalities. In one time step we evolve both main modes and the tail, and we check suitable consistency conditions, which guarantee that we produce valid bounds for solution of dissipative PDE under consideration.

List of participants

1) Arai, Zin, Kyoto University, Japan
2) Arioli, Gianni, Politecnico di Milano
3) Barrio, Roberto, University of Zaragoza, SPAIN
4) Barutello, Vivina, Università di Milano-Bicocca
5) Batko, Bogdan, WSB-NLU
6) Benedicks, Michael, Royal Institute of Technology
7) Berz, Martin, Michigan State University
8) Capiński, Maciej, Jagiellonian University, Poland
9) Celletti, Alessandra, Universitá di Roma &Tor Vergata& (Italy)
10) Chierchia, Luigi, Università ROMA TRE
11) Collins, Pieter, Centrum voor Wiskunde en Informatica
12) Ćwiszewski, Aleksander, N. Copernicus Univ., Toruń
13) Derivière, Sara, Universite de Sherbrooke, Canada
14) Downarowicz, Tomasz, Politechnika Wrocławska, Wrocław, Poland
15) Duda, Jarosław, Jagiellonian University, Poland

16) Ekola, Tommy, KTH Mathematics
17) Fura, Justyna, Nicolaus Copernicus University, Poland
18) Galias, Zbigniew, AGH, Kraków
19) Garay, Barnabas, Budapest Univ. of Technology
20) Gawrycka, Joanna, N. Copernicus University, Toruń, Poland
21) Ghrist, Robert, University of Illinois, Urbana
22) Gołębiewska, Anna, UMK Toruń, Poland
23) Hampton, Marshall, University of Minnesota Duluth
24) Han, Sejin, University of Maryland
25) Hell, Juliette, Freie Universität Berlin
26) Johnson, Tomas, Uppsala University
27) Junge, Oliver, Munich University of Technology
28) Kaczynski, Tomasz, Université de Sherbrooke
29) Kalies, William, Florida Atlantic University
30) Kapela, Tomasz, Jagiellonian University, Kraków
31) Kemper, Jens, University of Bielefeld, Bielefeld, Germany
32) Kiss, Gábor, University of Szeged
33) Kokubu, Hiroshi, Kyoto University, Japan
34) LaValle, Steven M., University of Illinois
35) Lessard, Jean-Philippe, Georgia Tech
36) de la Llave, Rafael, U. Texas at Austin
37) Lust, Alexander, University of Bielefeld, Bielefeld, Germany
38) Luzzatto, Stefano, Imperial College London
39) Maier-Paape, Stanislaus, RWTH Aachen
40) Makino, Kyoko, Michigan State University
41) Maksymenko, Sergiy, NAS of Ukraine, Kiev, Ukraine
42) Marzantowicz, Wacław, UAM, Poznań, POLAND
43) Mischaikow, Konstantin, Georgia Tech / Rutgers Univ.
44) Mrozek, Marian, Jagiellonian Univ. and WSB-NLU, Poland
45) Muchewicz, Krzysztof, N. Copernicus University, Toruń, Poland
46) Mączyńska, Zofia, Jagiellonian University, Poland
47) Newhouse, Sheldon, Michigan State University
48) Nishiura, Yasumasa, RIES, Hokkaido University
49) Nowicki, Tomasz, IBM
50) Oka, Hiroe, Ryukoku University, Japan
51) Oprocha, Piotr, AGH, Kraków, Poland
52) Pilarczyk, Paweł, Jagiellonian Univ. & Georgia Tech
53) Pochinka, Olga, Russia
54) Pokojska, Eva, Uni S. Bohemia, Czech Republic
55) Reineck, James, SUNY at Buffalo
56) Rybicki, Sławomir, N. Copernicus University, Toruń, Poland
57) Sakajo, Takashi, Hokkaido Univ.
58) Simó, Carles, Universitat de Barcelona
59) Srzednicki, Roman, Jagiellonian University
60) Stoffer, Daniel, ETH Zurich
61) Sędziwy, Stanisław, Jagiellonian University, Poland

62) Tabor, Jacek, Jagiellonian University
63) Terracini, Susanna, Universita' di Milano Bicocca
64) Tucker, Warwick, Uppsala University
65) Wanner, Thomas, George Mason University
66) Wilczak, Daniel, Jagiellonian University
67) Wilczyński, Paweł, Jagiellonian University, Poland
68) Wójcik, Klaudiusz, Jagiellonian University
69) Wójcik, Wojciech, Vrije Universiteit Amsterdam, NL
70) Zalewski, Mikołaj, Jagiellonian University
71) Zgliczynski, Piotr, Jagiellonian University, Krakow, Poland
72) Żelawski, Marcin, Jagiellonian University, Poland
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