

Experiments with the package conley

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(Joint work with Stanislaus Maier-Paape)

- 1 An Example
 - An Example of Franzosa
 - A Cahn-Hilliard example

- 2 The package conley
 - The package conley

Overview

1

An Example

- An Example of Franzosa
- A Cahn-Hilliard example

2

The package conley

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The equation

This simple example is taken from [Fra89]:

A 2-dimensional dynamical system with parameter θ

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \theta y - x \left(x - \frac{1}{3} \right) (1 - x). \end{aligned} \tag{1}$$

The equilibria

Three θ -independent equilibria

$$M(1) = M_{\theta}(1) = \left\{ \left(\frac{1}{3}, 0 \right) \right\},$$

$$M(2) = M_{\theta}(2) = \{(0, 0)\},$$

$$M(3) = M_{\theta}(3) = \{(1, 0)\}.$$

The abstract flow for general θ

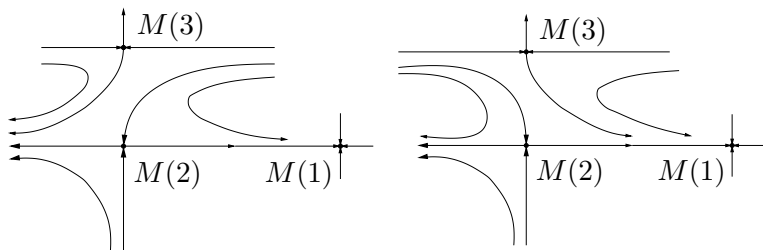


Figure: Abstract flow for $0 < \theta' \ll 1$ (left) and $\theta'' \gg 1$ (right)

The abstract flow for special θ

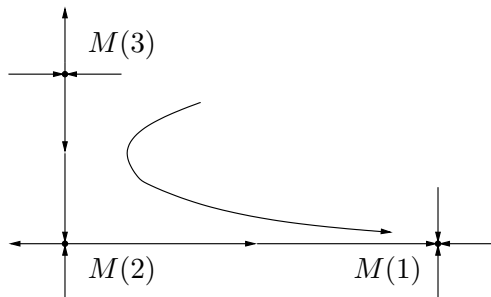


Figure: Special (abstract) flow at the bifurcation point θ_{special}

The Conley indices

The θ -independent Conley indices

$$\begin{aligned}CH_*(M(1)) &= \Sigma^0, \\CH_*(M(2)) &= \Sigma^1, \\CH_*(M(3)) &= \Sigma^1.\end{aligned}$$

The associated transition equation

A gradient flow φ serving as a transition system connecting the two systems (1) at $\theta = \theta'$ and $\theta = \theta''$:

The associated 3-dimensional dynamical system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \theta y - x \left(x - \frac{1}{3} \right) (1 - x) \\ \dot{\theta} &= \varepsilon(\theta' - \theta)(\theta'' - \theta), \end{aligned} \tag{2}$$

The abstract flow of the transition system

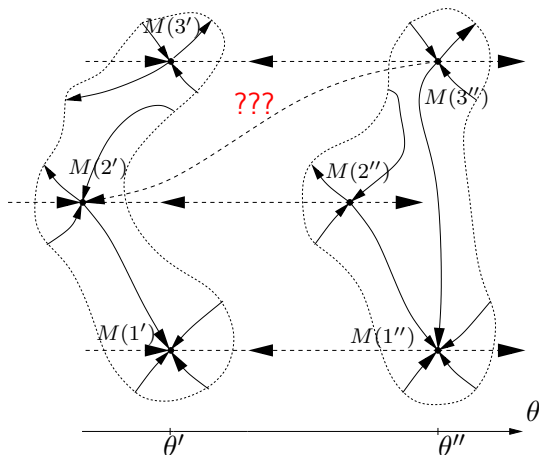
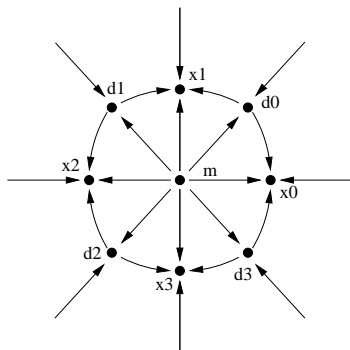


Figure: The flow φ of the transition system

The abstract flow of some Cahn-Hilliard equation

Recall from Stani's talk: The symmetry of Cahn-Hilliard equation in some parameter range



is the dihedral group D_8 [MPMW07].

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A homalg based package

conley is a Maple package (based on the the package homalg), which is able to compute connection and transition matrices under presence of constraints and of symmetries.

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We will now use the package conley to prove the existence of the connecting orbit indicated by ???

The abstract flow of the transition system

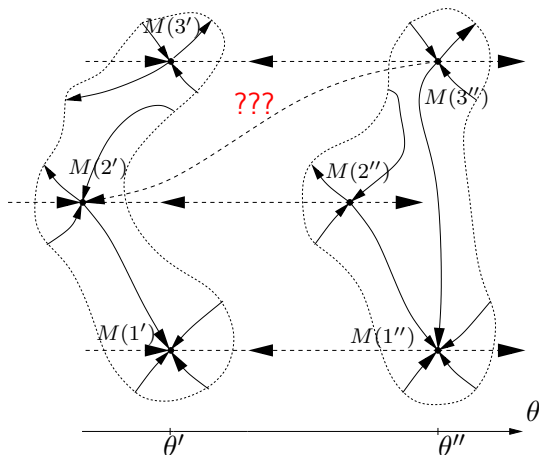
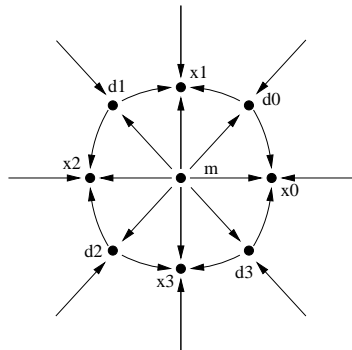



Figure: The flow φ of the transition system


The abstract flow of some Cahn-Hilliard equation


The symmetry of Cahn-Hilliard equation in some parameter range




is the dihedral group D_8 .

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Mohamed Barakat and Stanislaus Maier-Paape, *Computation of connection matrices using the software package conley*, Internat. J. Bifur. Chaos Appl. Sci. Engrg., accepted for publication (<http://wwwb.math.rwth-aachen.de/conley>).
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Mohamed Barakat and Daniel Robertz, *conley – computing connection matrices in Maple*, J. Symb. Comp. **44** (2009), 540–557, (arXiv:math.DS/0701173).
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Robert D. Franzosa, *The connection matrix theory for Morse decompositions*, Trans. Amer. Math. Soc. **311** (1989), no. 2, 561–592. MR MR978368 (90a:58149)
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Stanislaus Maier-Paape, Konstantin Mischaikow, and Thomas Wanner, *Structure of the attractor of the Cahn-Hilliard equation on a square*, Internat. J. Bifur. Chaos Appl. Sci. Engrg. **17** (2007), no. 4, 1221–1263. MR MR2329522