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### Spectral stability for the wave equation with periodic forcing

We consider the spectral stability problem for Floquet-type systems such as the wave equation $v_{tt} = \gamma^2 v_{xx} - \psi v$ with periodic forcing $\psi$. Specific results are obtained for a system where the forcing is due to a coupling between the wave equation and a time-period solution of a nonlinear beam equation. We prove (spectral) stability for some period and instability for another.

Ferenc Agoston Bartha
University of Szeged

### Global stability in a system using echo for position control

We consider a system of equations describing automatic position control by echo. The system can be reduced to a single differential equation with state-dependent delay. The delayed terms come from the control mechanism and the reaction time. H.-O. Walther proved that stable periodic motion is possible for large enough reaction time. We show that, for sufficiently small reaction lag, the control is perfect, i.e., the preferred position of the system is globally asymptotically stable.

Bogdan Batko
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### Conley index approach to sampled dynamics. Part I

There are many dynamical systems for which neither analytic computations nor a rigourous numerical analysis are possible, and sampling the dynamical system is the only way to infer some knowledge about the system. Then, there is the question whether the finite amount of experimental data is sufficient to obtain some global, general knowledge about the system and how to do it. In general, this is difficult, particularly in the case of chaotic dynamics. In this situation the coarseness of topological invariants turns out to be helpful. In particular, it is demonstrated in [5, 6] that the Conley index combined with multivalued approach suffices to detect chaotic dynamics in experimental data.

In the talk we describe the construction of a multivalued upper semicontinuous map from experimental data. Although such a map need not be acyclic, it still may induce a map in homology. For this it suffices to construct its homologically consistent enlargement $F$ which is homologically complete (cf. [3]). The problem with such an $F$ is that it need not admit any continuous selector. Fortunately, we may use Conley theory for such maps developed recently in [2, 1, 4].

We focus on the structure of an isolated invariant set $S$ with respect to $F$. Since isolating neighborhoods for multivalued maps do not necessarily admit index pairs (cf. [2]), we work with weak index pairs. We present accurate results to detect orbits passing through the disjoint components of $S$ in a given fashion. Conditions that guarantee their existence are expressed in terms of the index map $I_P$ associated with a weak index pair $P$ for $S$. Applying the Lefschetz-type fixed point theorem (cf. [7]) we provide sufficient conditions for the periodicity of such orbits. Moreover, we discuss the semiconjugacy with a shift dynamics of finite type.

Algorithms and actual applications of our approach will be presented in the second part of the talk, by M. Przybylski.

This is a joint work with K. Mischaikow, M. Mrozek and M. Przybylski.

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## Emergence of non-ergodic, conservative dynamics

With two works in progress: one with Jairo Bochi and one with Dimitry Turaev.

The Birkhoff ergodic theorem states that given an ergodic probability measure $\mu$, for $\mu$-almost every point $x$, the Birkhoff average:

$$S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$$

converges to $\mu$. For many differentiables maps $f$, there are finitely many ergodic probability measures $(\mu_i)_i$ so that for Lesbesgue a.e. $x \in M$, $S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$ converges to one of the $\mu_i$. Interestingly, this class of systems contains some of those with positive entropy (system for which it is impossible to predict the precise position of many orbits).

However, it has been shown in [Be16], that this paradigm is not sufficient to describe many typical dynamical systems. More precisely, we showed that among some open subsets of differentiable dynamical systems of a compact manifold $M$, typically in the sense of Kolmogorov, a system displays infinitely many attractors with very different statistical behaviors. To describe the complexity of such dynamics, the following notion has been introduced in [Be17]:

Given a standard distance on the probability measures of a space (such as the Wasserstein metric), the Emergence $\mathcal{E}(\epsilon)$ at scale $\epsilon > 0$ of a system is the minimal number $N$ of probability measures $(\mu_i)_{1\le i\le N}$ necessarily so that the Birkhoff average $S_k(x) := \frac{1}{k}\sum_{i=1}^{k}\delta_{f^i(x)}$ satisfies:

$$\limsup_{k\to\infty}\int_M d_{W_1}(S_k(x), \{\mu_i : 1\le i\le N\})dLeb < \epsilon\ ,$$

where $d_{W_1}$ is the 1-Wasserstein metric on the space of probability measures of $M$.

In [Be17], it has been conjectured:

In many categories of dynamical systems, a typical dynamics displays a super polynomial emergence:

$$\limsup_{\epsilon\to 0}\frac{\log\mathcal{E}(\epsilon)}{-\log\epsilon} = \infty\ .$$

We will present recent developments on this program, and its analog with the theory of systems of positive entropy (including the positive entropy conjecture [BT17]).

In a work in progress with Jairo Bochi, we showed an analog of the variational principle of the entropy for the concept of emergence. Furthermore, we showed that in the open set of conservative, surface mapping diffeomorphisms displaying an elliptic point, a $C^\infty$-generic diffeomorphism displays a maximal emergence (which is super-polynomial):

$$\limsup_{\epsilon\to 0}\epsilon^2\log\mathcal{E}(\epsilon) > 0\ .$$

In a work in progress with D. Turaev, we showed that in the open set of Hamiltonian diffeomormhisms with a totally elliptic point, a typical diffeomorphism in the sense of Kolmogorov (i.e. Lebesgue a.e. map in a generic family) displays a maximal emergence:

$$\limsup_{\epsilon\to 0}\epsilon^{2n}\log\mathcal{E}(\epsilon) > 0\ .$$

This proves this conjecture in the category of Hamiltonian (and surface, conservatif) context displaying an elliptic fixed periodic point.

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Bartosz Bieganowski
Nicolaus Copernicus University

**The variational setting on the Nehari manifold for functionals with a sign-changing nonlinear part**

Our aim is to provide an abstract, variational setting which allows us to deal with partial differential problems with sign-changing nonlinearities.

We consider equations of the form

$$Lu = f(x,u), \quad u \in E,\ x \in \mathbb{R}^N$$

where $f(x,u) = f_1(x,u) - f_2(x,u)$. We say that a function is a solution if it is a critical point of a nonlinear functional $\mathcal{J} : E \to \mathbb{R}$ of class $\mathcal{C}^1$, where $E$ is an appropriate function space. We define the Nehari manifold

$$\mathcal{N} := \{u \in E \setminus \{0\} \ : \ \mathcal{J}'(u)(u) = 0\}.$$

Obviously $\mathcal{N}$ contains all critical points of $\mathcal{J}$. We show that, under suitable conditions on $\mathcal{J}$, that $\mathcal{N}$ is homeomorphic with the unit sphere $S$ in $E$ and using this homeomorphism we can find a minimizing sequence.

At the end we would like to present applications of our variational theorem to the Schrödinger (also nonlocal) equations with sign-changing nonlinearities like

$$-\Delta u + V(x)u = f(x,u) - \Gamma(x)|u|^{q-2}u$$

or

$$(-\Delta)^{\alpha/2}u + V(x)u = f(x,u) - \Gamma(x)|u|^{q-2}u$$

for bounded, close-to-periodic potentials $V$ and also for singular potentials like $V(x) = -\frac{\mu}{|x|^\alpha}$.

Maxime Breden
Technical University of Munich

**Computer assisted proof for the Navier-Stokes equations: existence of periodic orbits in a Taylor-Green flow**

In this talk, I will explain how computer-assisted methods can be used to study the forced, incompressible Navier-Stokes equations, and in particular to prove the existence of periodic orbits. We work with the vorticity formulation, for which we express the periodic solutions as space-time Fourier series. We start by numerically solving for the Fourier coefficients of the vorticity. Then, we prove the existence of an exact solution in a neighborhood of the numerically computed approximation (in a well chosen Banach space of algebraically decaying coefficients). This is done via a Newton-Kantorovich type of argument, combining analytic estimates and the usage of interval arithmetic. The high dimensionality of the problem is softened by the many symmetries present in the solutions, which allows us to reduce the number of Fourier coefficients to a manageable level.

We illustrate the applicability of our method by proving the existence of periodic orbits for the Navier-Stokes equations in a Taylor-Green flow, which is an important example to understand the transition to turbulence.

This is joint work with Jan Bouwe van den Berg, Jean-Philippe Lessard and Lennaert van Veen.

Florent Brehard
ENS de Lyon & LAAS-CNRS

**A computer assisted proof for a new lower bound on H(4) in Hilbert's 16th problem**

We provide a computer-assisted proof for a new lower bound on H(4) in the Hilbert 16th problem, that is the maximum number of limit cycles that can occur in a polynomial planar vector field of degree 4. Indeed, we exhibit a quartic vector field for which we rigorously prove the existence of at least 24 limit cycles.

Hilbert's 16th problem is part of Hilbert's famous list of 23 problems presented in 1900 at the International Congress of Mathematicians in Paris. The second part of this problem asks whether there exists, for each natural integer n, a finite upper bound H(n) on the number of limit cycles a polynomial planar vector field of degree n can have. For now, a proof of this conjecture seems out of reach, even for n = 2. A simpler version of this problem restricts the investigation to perturbed Hamiltonian systems.

The upper bound Z(n) for such systems of degree n was proved to be finite for all n. A key ingredient is the Poincaré-Pontryagin theorem that relates the number of limit cycles with the number of zeros of so-called Abelian integrals along the level curves of the potential function associated to the Hamiltonian system.

The quartic system we investigate is not Hamiltonian but still integrable, so that the Poincaré-Pontryagin theorem still applies. Our work consists in approximating the Abelian integral associated to each monomial of the perturbation (as function of the energy level of the potential function), adjusting the coefficients to maximize the number of zeros of the resulting linear combination, and finally computing rigorous enclosures of the integral at various points to certify the number of sign changes. For that purpose, we use Rigorous Polynomial Approximations [1,2,3] via our free C library available here [4], which allows us to perform rigorous and efficient evaluations of the Abelian integral.

A formalization of this result in Coq is in progress, in order to avoid possible implementation errors that may lead to a wrong result.

This is a joint work with Nicolas Brisebarre and Warwick Tucker.

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## Arnold Diffusion with Quantitative Estimates

We develop a topological method to show the existence of Arnold diffusion in a class of a priori chaotic, autonomous Hamiltonian systems subject to small, time-periodic perturbations. We show that there exists orbits along which the energy (action) changes by order one with respect to the perturbation parameter, as well as orbits for which the energy exhibits symbolic dynamics. We also obtain quantitative information on the diffusion orbits: diffusion speed, explicit range of perturbation parameters and Hausdorff dimension of the set of initial conditions that undergo symbolic dynamics. A main feature of our topological method is that it can be implemented in computer assisted proofs. As an application, we provide an explicit construction of diffusing orbits in a concrete model of the planar elliptic restricted three-body problem, describing the motion of an asteroid relative to the Neptune-Triton system.

Chiara Caracciolo
University of Rome

## Elliptic tori in FPU chains

We revisit an algorithm constructing elliptic tori, that was originally designed to apply to planetary problems. The scheme is adapted to properly work with models of chains of $N + 1$ particles interacting via anharmonic potentials, thus covering also the case of FPU lattices. After having preliminarly settled the Hamiltonian in a suitable way, we perform a sequence of canonical transformations removing the undesired perturbative terms by an iterative procedure. This is done by using the Lie series approach, that is explicitly implemented in a programming code with the help of a software package, specially designed for computer algebra manipulations. We successfully apply our new algorithm to the construction of 1D-dimensional elliptic tori for wide sets of the parameter (i.e. the total energy of the system) that rules the size of the perturbation in FPU chains with $N = 4$, 8. Moreover, we show the stability of regions surrounding the elliptic 1D-tori. We compare our semi-analytical results with those provided by numerical explorations of the FPU-model dynamics, that are made by using frequency analysis methods. We find that our procedure works up to values of the total energy that are of the same order of magnitude with respect to the maximal ones, for which elliptic tori are detected by the numerical methods.

Jose Gabriel Carrasquel Vera
Adam Mickiewicz University of Poznań

### The new rational homotopy theory applied to sectional categories

In this talk we will develop some new rational homotopy theory techniques and show how they can be used to estimate sectional categories such as topological complexity and Lusternik-Schnirelmann category through the prism of Quillen models. We will also show how this approach can be used to address some open problems.

Joint work with Urtzi Buijs and Lucile Vandembroucq.

Marc Ethier
Université de Saint-Boniface

### A new theoretical approach to the comparison of 2D persistence diagrams in sublevel set persistent homology

Two-dimensional sublevel set persistent homology for a filtering function $f : X \to \mathbb{R}^2$ is known to reduce to a parametrized family of functions $f^*_{(a,b)} : X \to \mathbb{R}$ where $a \in ]0,1[$, $b \in \mathbb{R}$. For two filtering functions $f$ and $g$, the *two-dimensional matching distance* $D_{\text{match}}(f,g)$ is computed by taking the supremum of the classical bottleneck distance between the persistence diagrams $\text{Dgm}\left(f^*_{(a,b)}\right)$ and $\text{Dgm}\left(g^*_{(a,b)}\right)$ over $(a,b)$. While it was observed in experiments that this supremum was often reached at a value of $a$ equal to or close to $1/2$, the lack of geometrical properties in the definition of $D_{\text{match}}$ precluded a proof of this result. Considering only matchings between $\text{Dgm}\left(f^*_{(a,b)}\right)$ and $\text{Dgm}\left(g^*_{(a,b)}\right)$ that change continuously with respect to $(a,b)$ led to the definition of the *coherent matching distance* $CD_U(f,g)$. The goal of this talk is to define this distance as the minimum *coherent cost* over matchings $\sigma_{(a,b)}$ between $\text{Dgm}\left(f^*_{(a,b)}\right)$ and $\text{Dgm}\left(g^*_{(a,b)}\right)$, where the basepoint $(a,b)$ is arbitrary, transported along a path $\pi$ starting at $(a,b)$ over a well-chosen open subset $U$ of parameter values. We culminate with a proof that the coherent cost is always reached on a path $\pi$ whose end belongs to the line $a = 1/2$ or to the boundary of $U$, strongly suggesting that computing $CD_U$ will allow one to manage the parameter space $]0,1[\times\mathbb{R}$ more efficiently.

Leonid Fedorov
MPI for Biological Cybernetics

### Identifying charachteristic waveforms without prior shape knowledge in LFP signal: preliminary observations from spontaneous brain activity

Local Field Potential (LFP) is an electrophysiological signal summarizing synaptic and somato-dendritic currents. As such, it depends on a spatial orientation of pyramidal neurons around the electrode and distance to them, and can be treated as a summary of a fine-grained spiking activity signal. In subcortical structures: e.g. Pontine Nuclei, Thalamus and regions of Hippocampus, structure-specific waveforms have been studied, named p-waves, spindles, sharp-wave ripples and more, each given it's characteristic shape. Prior methods of their identification relied on spectral-analysis of LFP time series, adopted to knowing the behavioral state of an animal (e.g. stages of sleep: REM, slow-wave sleep etc.).

Here we attempt an approach to the inverse problem: identifying repetitive waveforms in LFP signal without a-priori knowledge about their shape or behavioral state of the animal. This will allow detection of fine grained behavioral state transitions, especially in wakefullness, and also give a reference for temporal segmentation of brain activity measured with methods at larger global scale, e.g. BOLD imaging.

We only assume that any candidate waveform must repeat consecutively to be considered an indicator of a brain state. To capture it, we construct a point-cloud $X$ by taking overlapping windows of variable length from the time series. We thus expect that a consecutively repeating waveform, of arbitrary unknown shape, if exists, will manifest itself in such point cloud as a sample from $S^1$ (albeit embedded in $\mathbb{R}^d$, where $d$ is the number of original time series points within a window of (re-normalized) length). We then look at an approximate persistence landscape using bootstrap [F. Chazal et al., *Subsampling Methods for Persistence Homology*, PMLR 37:2143-2151, 2015] to capture the persistent 1-st homology groups in the point cloud.

Critically, we metrize the point cloud of waveforms $X$ by learning it's partially scale- and shift-invariant representation using a convolutional autoencoder $\Pi$ of the form:

$$\begin{aligned}\Pi(X) &= \Phi_n \circ ... \circ \Phi_0(X),\\ \Phi_k(X) &= ReLU(\Psi(\Phi_{k-1}(X) * W_k + b_k)),\end{aligned}$$

with nonlinear operation $ReLU(x) = max(x, 0)$, convolution operator $*$ with weight matrix $W_k$ and bias term $b_k$ at layer $k$, and a norm-pooling operation $\Psi$. In addition to accounting for biological sources of noise that may deform an idealized waveform, the autoencoder conveniently lets choose a metric $\rho$ on $X$ naively. Then the persistent homology groups in the point-cloud $X$ are looked for with a candidate list of learnt metrics $\rho_k(x, y) = \rho(\Phi_k(x), \Phi_k(y))$ taken from evaluating the autoencoder on $X$ and considering response at layer $k$ (where the original dimension $d$ can be changed altering autoencoder architecture).

Marco Fenucci
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**On the stability of periodic N-body motions with the symmetry of Platonic polyhedra**

In (Fusco et. al., 2011) several periodic orbits of the Newtonian $N$-body problem have been found as minimizers of the Lagrangian action in suitable sets of $T$-periodic loops, for a given $T > 0$. Each of them share the symmetry of one Platonic polyhedron. In this talk we first present an algorithm to enumerate all the orbits that can be found following the proof in (Fusco et. al., 2011). Then we describe a procedure aimed to compute them and study their stability. Our computations suggest that all these periodic orbits are unstable. For some cases we produce a computer-assisted proof of their instability using multiple precision interval arithmetic.

Valery Gaiko
NAS Belarus

**Geometric and topological aspects of global bifurcation theory for polynomial dynamical systems**

We develop geometric and topological aspects of bifurcation theory for studying polynomial dynamical systems. It gives a global approach to the qualitative analysis of such systems and helps to combine all other approaches, their methods and results. We carry out the global bifurcation analysis of multi-parameter systems. To control all their limit cycle bifurcations, especially, bifurcations of multiple limit cycles, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of new bifurcational geometric methods based on Perko's planar termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by A. Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and used Puiseux series to show that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. Such a bifurcation can happen, e. g., in a three-dimensional Lorenz system. But this cannot happen for planar systems. That is why the Wintner–Perko termination principle is applied for studying multiple limit cycle bifurcations of planar polynomial dynamical systems. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Applying this approach, we have solved, e. g., *Hilbert's Sixteenth Problem* on the maximum number and distribution of limit cycles for the general Liénard polynomial system with an arbitrary number of singular points and for the Kukles cubic-linear system. We have also applied this approach for studying global limit cycle bifurcations of Holling-type systems which model the population dynamics in biomedical and ecological systems. Finally, applying a similar approach, we have considered various applications of three-dimensional polynomial dynamical systems and, in particular, completed the strange attractor bifurcation scenario in the classical Lorenz system globally connecting the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.

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## A rigorous enclosure of the cubic Chua's attractor

In this work, we prove the existence of a trapping region enclosing the double scroll attractor for the Chua's circuit with a cubic nonlinearity. This attractor contains an unstable equilibrium, and typical trajectories belonging to the attractor pass arbitrarily close to this equilibrium. Normal form theory is used to develop a method to find enclosures of trajectories in a neighborhood of an unstable fixed point of a spiral type. This is a joint work with Warwick Tucker, Department of Mathematics, Uppsala University, Uppsala, Sweden.

The dynamics of the Chua's circuit with a cubic nonlinearity [1] is defined by

$$\begin{aligned} C_1\dot{x}_1 &= (x_2 - x_1)/R - g(x_1), \\ C_2\dot{x}_2 &= (x_1 - x_2)/R + x_3, \\ L\dot{x}_3 &= -x_2 - R_0x_3. \end{aligned} \tag{1}$$

This dynamical system is symmetric with respect to the transformation $(x_1, x_2, x_3) \mapsto (-x_1, -x_2, -x_3)$. The following values of parameters are considered: $C_1 = 0.7$, $C_2 = 7.8$, $L = 1.891$, $R_0 = 0.01499$, $g_1 = -0.59$, $g_2 = 0.02$, and $R = 2.0$. For these parameter values the double scroll attractor is observed in simulations.

The origin $\bar{x} = (0, 0, 0)$ is an unstable equilibrium of (1). For this equilibrium the Jacobian matrix $J$ has one real eigenvalue $\lambda \approx 0.2066098948$ and a pair of complex eigenvalues $\alpha \pm \beta i \approx -0.0750340265 \pm 0.1965518222i$.

Let $B$ be a transformation matrix converting the linear system $\dot{x} = Jx$ to the linear system $\dot{y} = Dy$ with the matrix $D$ in the Jordan normal form

$$D = B^{-1}JB = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & -\beta & \alpha \end{pmatrix},$$

$$B \approx \begin{pmatrix} -0.96026919 & 0.67812191 & 0 \\ -0.10491311 & -0.19329705 & 0.18660054 \\ 0.25860457 & -0.60865747 & -0.31225509 \end{pmatrix}.$$

Let $\Sigma = \Sigma_1 \cup \Sigma_2$ be the union of two planes $\Sigma_1 = \{x\colon x_1 = 2.1647\}$ and $\Sigma_2 = \{x\colon x_1 = -2.1647\}$. The *return map* $P : \Sigma \mapsto \Sigma$ is defined as $P(x) = \varphi(\tau(x), x)$, where $\varphi(t, x)$ is the trajectory of (1) based at $x$, and $\tau(x)$ is the *return time* after which the trajectory $\varphi(t, x)$ returns to $\Sigma$.

A candidate $T = T_1 \cup T_2 \cup T_3 \cup T_4$ for a trapping region enclosing the numerically observed attractor has been constructed in [2]. $T_3, T_4 \subset \Sigma_2$ are symmetric to $T_1, T_2 \subset \Sigma_1$. In [2], it was shown that standard rigorous integration procedures fail to evaluate the return map over the whole set $T$. This is a consequence of the fact that the double scroll attractor contains the origin—an unstable equilibrium. Some trajectories remain close to each other until they come close to the origin where they separate and go in opposite directions. A rigorous evaluation of the return map over sets containing discontinuities requires handling trajectories passing arbitrarily close to an equilibrium. Such trajectories have infinitely large return times. For linear systems the stable manifold is flat and explicit formulas for solutions can be used [3].

In this work using rigorous integration methods and the normal form theory [4] we prove the following theorem.

**Theorem 1** For each $x \in T$ either $P(x) \in T$ or the trajectory $\varphi(t, x)$ with the initial point $x$ converges to the origin without intersecting $\Sigma$, i.e., $\varphi(t, x) \to (0, 0, 0)$ for $t \to \infty$ and $\{\varphi(t, x)\colon t > 0\} \cap T = \emptyset$.

From the symmetry of the problem it is sufficient to verify the assertions for the set $T_1 \cup T_2$.

First, we prove that $P(T_2) \subset T_1$. It is sufficient to prove that the image of the border of $T_2$ is enclosed in $T_1$ and that the return map is well defined on $T_2$. The CAPD library [5] was used for a rigorous evaluation of the return map. During the proof the generalized bisection procedure was used. The border was covered by 1267 boxes, and the interior was covered by 4513 boxes. Computations regarding the border and the interior took 16 and 58 seconds, respectively, using a single-core 3.1 GHz processor.

To carry out the proof for the set $T_1$, we construct a cylinder centered at the origin spanned by the eigenvectors of $J$. Let us denote by $C_y(h, r) = \{y\colon |y_1| \le h, y_2^2 + y_3^2 \le r^2\}$ the cylinder centered at the origin with the height $2h > 0$ and the radius $r > 0$. Let $C_x(h, r) = BC_y(h, r)$ denote the cylinder $C_y(h, r)$ transformed by the change of coordinates $x = By$. The axis of $C_x(h, r)$ is the unstable one-dimensional eigenspace of the origin and bases of $C_x(h, r)$ are parallel to the stable eigenspace of the origin.

The proof regarding the set $T_1$ consists of three steps. In the first step, we prove that for each $x \in T_1$ either $P(x) \in T$ or the trajectory started at $x$ enters the cylinder $C_x(h,r)$ through the entry set. The *entry set* is the side of the cylinder $C_x(\bar{h},r)$ with $\bar{h} = 0.02$. In the second step, we show that trajectories starting in the entry set either converge to the origin of leave the cylinder through the exit set. The *exit set* is composed of two bases of the cylinder $C_x(h,\bar{r})$ with $\bar{r} = 0.075$. In the third step, we prove that all trajectories starting in the exit set reach the trapping region $T$.

To carry out the first step, similarly as for the set $T_2$ the interior and the border are handled separately and the method of generalized bisection is used. For a given box $\mathbf{v}_k$, we attempt to find an enclosure $\mathbf{w}_k$ of $P(\mathbf{v}_k)$ and verify that $\mathbf{w}_k \subset T$. If this is not successful, we try to evaluate the return map $P_{\text{cyl}}$ with the section being the side of the cylinder $\{(x_1,x_2,x_3)\colon y_2^2+y_3^2 = r^2, |y_1| \le \bar{h} \text{ where } y = B^{-1}x\}$. If any of these two conditions is verified then the computations regarding the box $\mathbf{v}_k$ are finished. Otherwise, the generalized bisection is used to split $\mathbf{v}_k$ into smaller boxes to be processed. During these computations, the border of $T_1$ was covered by 6228 boxes. For 6220 boxes the condition $P(\mathbf{v}_k) \subset T$ was verified. For the remaining 28 boxes it has been shown that the return map $P_{\text{cyl}}$ is well defined. The computations took 148 seconds.

The most time-consuming part of the proof is to show that return maps are well-defined on $T_1$. During the proof the set $T_1$ was covered by 1012118 boxes. For 698010 boxes it was shown that the return map $P$ is well-defined and for the remaining 314108 boxes it was shown that the return map $P_{\text{cyl}}$ is well-defined. The computations took approximately 17 hours.

In the second step, we show that all trajectories starting in the entry set either converge to the origin or reach the exit set. This is done using the normal form theory. We construct a nonlinear change of coordinates in such a way that the nonlinear vector field (1) is transformed into an *almost linear* vector field $\dot{y} \approx Dy$ inside the cylinder. We compute the perturbations which allow us to compute bounds for the solutions of the nonlinear vector field from the results obtained for the diagonal linear vector field. There are three types of perturbations which need to be considered: the entry perturbation $\varepsilon_{\text{entry}}$, the exit perturbation $\varepsilon_{\text{exit}}$, and the eigenvalue perturbation $\varepsilon_{\text{eig}}$. To compute an enclosure of a trajectory passing through the cylinder we first perturb the entry set by applying the entry perturbation, then find an enclosure of the exit set for the linear vector field $\dot{y} \approx Dy$ with eigenvalues perturbed by $\varepsilon_{\text{eig}}$, and finally apply the exit perturbation to obtain an enclosure of the exit set for the nonlinear vector field. Perturbations depend on the cylinder size. For the cylinder $C_x(h,r)$ with $h = 0.1$ and $r = 0.1$, we obtain the entry perturbation $\varepsilon_{\text{entry}} = 0.00596$, the exit perturbation $\varepsilon_{\text{exit}} = 0.00888$, and the eigenvalue perturbation $\varepsilon_{\text{eig}} = 2.02 \times 10^{-13}$. Using these values we verify that trajectories of the nonlinear system starting in the entry set either converge to the origin of leave the cylinder through the exit set.

In the third step of the procedure, we prove that all trajectories starting in the exit set return to $T_2$ or $T_4$. During the computations, the border of one of the bases of the cylinder $C_x(h,\bar{r})$ is covered by 209 boxes and it is shown that their images under the return map $P$ are enclosed in $T_2$. The interior is covered by 8866 boxes and it is proved that the return map is well defined on these boxes. Computations in this step took 152 seconds. This completes the computer assisted proof of Theorem 1.

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Tingran Gao

University of Chicago


## Synchronization Problems: From Geometry to Learning

We develop a geometric framework, based on the classical theory of fibre bundles, to characterize the cohomological nature of a large class of synchronization-type problems in the context of graph inference and combinatorial optimization. In this type of problems, the pairwise interaction between adjacent vertices in the graph is of a non-scalar nature, typically taking values in a group; the consistency among these non-scalar pairwise interactions provide information for the dataset from which the graph is constructed. We model these data as a fibre bundle equipped with a connection, and consider a horizontal diffusion process on the fibre bundle driven by a standard diffusion process on the base manifold of the fibre bundle; the spectral information of the horizontal diffusion decouples the base manifold structure from the observed non-scalar pairwise interactions. We demonstrate an application of this framework on evolutionary anthropology.

Reza Ghamarshoushatri
University of Sherbrooke

### Towards Conley Index for combinatorial vector field on a cubical complex

Robin Forman introduced a combinatorial vector field on simplicial and cellular complexes. In order to design dynamical systems tools such as Conley index in Forman's setting, some necessary developments in definitions and new concepts were required. That has been done recently in a joint work by Batko, Kaczynski, Mrozek, and Wanner in the case of a combinatorial vector field defined on a finite simplicial complex. They also provided a link between Forman's vector field and multivalued map dynamics. I currently work on an analogy of that development for cubical complexes. This work may bring insight to understanding dynamics of multi-dimensional data.

Anna Gierzkiewicz
University of Agriculture, Krakow

### Chaos in Hyperion's rotation: a computer-assisted proof

The inner rotation of Saturn's moon Hyperion is often modeled by equations of an ellipsoidal satellite on a Keplerian orbit, with rotation axis perpendicular to its plane. The angle of rotation $\theta$ fulfills a three-dimensional ordinary differential equation

$$\begin{cases} \theta' = \phi \\ \phi' = -\frac{\omega^2}{2r^3}\sin 2(\theta - f) \\ f' = \frac{(1+e\cos f)^2}{(1-e^2)^{3/2}} \end{cases} . \tag{2}$$

The model is expected to be chaotic for large range of parameters $e$, $\omega$. The aim of the talk is to present a rigorous computer-assisted proof of chaos in its dynamics by the use of CAPD C++ library. The proof follows the general method from [G. Arioli and P. Zgliczyński, *Symbolic dynamics for the Hénon–Heiles Hamiltonian on the critical level*, J. Diff. Eq. 171(1) (2001), 173–202].

We study the Poincaré map $P$ on the 2-dim section $\{f = 0\}$. In short, we search for the hyperbolic stationary points of $P$ and transversal intersections of their stable and unstable manifolds. Then we determine some special sets (*h-sets*) with a loop of *covering relations* between them. The existence of such loop connecting two hyperbolic points proves rigorously the existence of a Smale horseshoe type set for $P$ and, consequently, the chaotic behaviour of the system (2).

Jorge Gonzalez
Florida Atlantic University

### High-order parameterization of invariant manifolds for parabolic partial differential equations on irregular spatial domains

We consider the problem of computing unstable manifolds for equilibrium solutions of parabolic PDEs posed on irregular spatial domains. Our approach is based on the parameterization method, a general functional analytic framework for studying invariant manifolds of dynamical systems. The method leads to an infinitesimal invariance equation describing the unstable manifold, which we solve recursively via power matching scheme. The recursive scheme leads to linear homological equations for the jets of the manifold which, along with equilibrium and eigenvalue/eigenfuntion problems, we solve using the finite element methods. One feature of the method is that we recover the dynamics on the manifold in addition to its embedding. We implement the method for some example problems on various two dimensional domains.

Marcel Guardia
Universitat Politecnica de Catalunya

### Transverse instability and growth of Sobolev norms near quasiperiodic tori for the 2D cubic NLS equation

The study of solutions of Hamiltonian PDEs undergoing growth of Sobolev norms $H^s$ (with $s \neq 1$) as time evolves has drawn considerable attention in recent years. The importance of growth of Sobolev norms is due to the fact that it implies that the solution transfers energy to higher modes.

Consider the defocusing cubic nonlinear Schrödinger equation (NLS) on the two-dimensional torus. The equation admits a special family of invariant quasiperiodic tori. These are inherited from the 1D cubic NLS (on the circle) by considering solutions that depend only on one variable. We show that, under certain assumptions and over sufficiently long time scales, these tori are transverally unstable in Sobolev spaces $H^s$ $(0 < s < 1)$. More precisely, we construct solutions of the 2D cubic NLS which start arbitrarily close to such invariant tori in the $H^s$ topology and whose $H^s$ norm can grow by any given factor.

This is a joint work with Z. Hani, E. Haus, A. Maspero and M. Procesi.

Wouter Hetebrij
VU Amsterdam

### The parameterization method for Center Manifolds

In this talk we will generalize the parameterization method to Center Manifolds. The parameterization method was introduced in a series of 3 papers, *The parameterization method for invariant manifolds I,II &III*, by Cabré, Fontich and De la Llave to prove the existence of (Un)Stable Manifolds.

We consider a discrete dynamical system on a Banach space $X$ given by a $C^n$ diffeomorphism $f : X \to X$. For a non-hyperbolic fixed point of $f$, we will propose a new method to prove the existence and smoothness of a locally conjugate dynamical system on the center subspace. Furthermore, we will give explicit bounds on the non-linear part of $f$ for which the conjugacy between the dynamical system on $X$ and the dynamical system on the center subspace will be global.

Finally, we will use the generalized parameterization method to prove the existence of a period doubling bifurcation in a Reaction Diffusion equation. In this application, we will give explicit bounds on the parameter region where there exists a period 2-orbit. Furthermore, we will prove the existence of a heteroclinic orbit between the period 2-orbit and the fixed point inside this parameter region.

This is joint work with Jan Bouwe van den Berg and Bob Rink.

Jonathan Jaquette
Rutgers, The State University Of New Jersey

### A proof of Jones' conjecture

The Wright/Jones conjecture asserts that the nonlinear delay differential equation known as Wright's equation has no slowly oscillating periodic solutions (SOPS) for $\alpha < \pi/2$, and a single, unique SOPS for each $\alpha > \pi/2$. This talk presents a computer-assisted proof of these conjectures, focusing in particular on the mesoscopic parameter regime $\alpha \in (\pi/2, 1.9]$. Furthermore, we show there are no isolas of periodic solutions to Wright's equation; all periodic orbits arise from Hopf bifurcations.

Mioara Joldes
LAAS-CNRS

### Spacecraft collision probabilities: a holonomic approach for moment problems

The increasing number of space debris in Low Earth Orbits constitute a serious hazard for operational satellites. In this talk, we discuss a mathematical modeling for the general case of collision probability computation for multiple encounters between such objects. This theoretical formulation is based on the measure theory framework and the related developments on the generalized moment problem.

Related to this problem, we firstly discuss the approximate, but efficient, computation of higher order moments of some "nice" positive measure, whose density wrt. Lebesgue measure is holonomic. This class of functions contains many elementary and special functions, and in short, can be characterized by sufficiently many partial differential and difference equations, both linear and with polynomial coefficients. The algebraic properties of such functions allow for efficient algorithms, like the so-called creative telescoping which computes linear differential equations satisfied by multiple integrals with parameters.

Secondly, we attempt to approximately reconstruct the support of the initial collision-prone states in a two-objects long-term encounter, by extending a method of J.B. Lasserre and M. Putinar which makes clever use of Stokes theorem. Given a finite number of numerically computed moments for a measure with holonomic density, and assuming a real algebraic boundary for the support, we discuss a preliminary result on an algorithmic method for obtaining the coefficients of a polynomial vanishing on this boundary. This is joint work with D. Arzelier, F. Bréhard, D. Gueho, J-B. Lasserre, A. Rondepierre and in collaboration with CNES (French Space Agency).

Piotr Kalita
Jagiellonian University

**On non-autonomously forced Burgers equation with periodic and Dirichlet boundary conditions**

We study the non-autonomously forced Burgers equation

$$u_t(x,t) + u(x,t)u_x(x,t) - u_{xx}(x,t) = f(x,t)$$

on the space interval $(0,1)$ with two sets of the boundary conditions: the Dirichlet and periodic ones. For both situations we prove that there exists the unique $H^1$ bounded trajectory of this equation defined for all $t \in \mathbb{R}$. Moreover we demonstrate that this trajectory attracts all trajectories both in pullback and forward sense. We also prove that for the Dirichlet case this attraction is exponential. Finally, we present some preliminary results on computer assisted construction of this trajectory based on Finite Element Method for the Dirichlet conditions. The results are obtained in a joint work with Piotr Zgliczyński.

Shane Kepley
Rutgers University

**Automatic computation and continuation of connecting orbits with applications to Hamiltonian systems**

A key ingredient for understanding global dynamics is the study of invariant sets and connections between them. In this talk we will discuss recent research on algorithms for automatically computing homoclinic/heteroclinic orbits for differential equations and continuation of these orbits with respect to multiple parameters.

The first step in the method is to compute local (un)stable manifolds for invariant sets of interest, and then obtain atlases parameterizing the global manifolds by analytic continuation. These atlases are "mined" to find connecting orbits, or to rule out their existence, yielding information about the global structure of the connecting dynamics. The major drawback in computing the global parameterizations is its high computational cost and it is infeasible to compute global parameterizations at many parameter values. Instead, a large ensemble of connecting orbits are computed at a single parameter value and we apply numerical continuation methods to obtain connections over a wide range of parameters.

As an application example, we will illustrate the method by computing connecting orbits between saddle-focus equilibria for a particular $N$-body problem. We will also discuss some interesting results for general Hamiltonian systems which follow from the existence of these connections. This is joint work with J.D. Mireles James.

Hans Koch
The University of Texas at Austin

**Validated numerical solutions for some semilinear elliptic equations on the disk (with Gianni Arioli)**

Starting with approximate solutions of the equation $-\Delta u = wu^3$ on the disk, with zero boundary conditions, we prove that there exist true solutions nearby. One of the challenges here lies in the fact that we need simultaneous and accurate control of both the (inverse) Dirichlet Laplacean and nonlinearities. We achieve this with the aid of a computer, using a Banach algebra of real analytic functions, based on Zernike polynomials. Besides proving existence and symmetry properties, we also determine the Morse index of the solutions.

Karolina Kropielnicka
Polish Academy of Sciences and University of Gdańsk

### Splitting methods for Schrödinger equations with time dependent potentials; many problems, many approaches.

In this talk I will present various numerical approaches for the linear Schrödinger equations with time dependent potentials. As it often happens, the methods should be closely correlated with properties of the problem. We suggest an application of asymptotic Zassenhauss decomposition in tandem with Munthe-Kaas-Owren basis for semiclassical scaling. In case of high oscillations of the potential we resort to simplified commutators in Magnus expansion and integrate the potential function in the very last stage of the algorithm. Magnus-Lanczos with simplified commutators are proposed in case of atomic scaling. Irrespectively of chosen regime, compact splittings are suggested for equations under the influence of laser matter. I will present numerical examples and compare these methods numerically.

This talk is based on results obtained with Philipp Bader, Arieh Iserles and Pranav Singh.

Sergey Kryzhevich
Saint-Petersburg State University

### Infinite invariant mesures and Pugh's Closing Lemma

This is a joint result with Prof. Eugene Stepanov. In the recent paper by D. Burago, S. Ivanov and A. Novikov, "A survival guide for feeble fish", it has been shown that a fish with limited velocity can reach any point in the (possibly unbounded) ocean provided that the fluid velocity field is incompressible, bounded and has vanishing mean drift. This result extends some known global controllability theorems though being substantially nonconstructive. We give a fish a different recipe of how to survive in a turbulent ocean, and show its relationship to structural stability of dynamical systems by providing a constructive way to change slightly the velocity field to produce conservative (in the sense of not having wandering sets of positive measure) dynamics. In particular, this leads to the extension of Ch. Pugh's closing lemma to incompressible vector fields over unbounded domains. The results are based on an extension of the Poincaré recurrence theorem to some $\sigma$-finite measures and on specially constructed Newtonian potentials.

Hieronim Kubica
AGH University of Science and Technology

### Persistence of normally hyperbolic invariant manifolds in the absence of rate conditions

We consider perturbations of normally hyperbolic invariant manifolds, under which they can loose their hyperbolic properties. We show that if the perturbed map which drives the dynamical system preserves the properties of topological expansion and contraction, then the manifold is perturbed to an invariant set. The main feature is that our results do not require the rate conditions to hold after the perturbation. In this case the manifold can be perturbed to an invariant set, which is not a topological manifold. Our method is not perturbative. It can be applied to establish invariant sets within a prescribed neighbourhood also in the absence of a normally hyperbolic invariant manifold prior to perturbation. The work is in the setting of nonorientable Banach vector bundles, without needing to assume invertibility of the map.

Jose Luis Licon Salaiz
University of Cologne

### Computational topology in the understanding of atmospheric turbulence

The atmospheric boundary layer is modeled as a convective system driven by buoyancy from the surface, in which the flow exhibits fully-developed turbulence. Coherent structures, defined as regions of the spacetime domain in which the flow has similar properties, e.g. convective plumes or thermals, play a significant role in the characterization of such a system. Our research focuses on the interaction of the atmospheric boundary layer with land-surface patterns of varying heterogeneity, an interaction we expect to characterize in terms of flow morphology. To this end we use numerical simulations generated by a mesoscale atmosphere-land model, LES-ALM (cf. Shao et al., 2013).

Current data analysis methods, when applied to these simulations, don't allow a direct representation and study of coherent structures. Working with the meteorologists behind the LES-ALM model we have applied techniques from computational topology to address this issue. More specifically: by implementing an augmented contour tree data structure we are able to isolate large scale features in the model outputs, describe their spatio-temporal evolution, and quantify their dependence on land-surface patterns. We also use persistent homology to quantify the degree of spatial organization of flow structures, and to study their relation with other mesoscale atmospheric phenomena, such as the spatial clustering of cumulus clouds. These topological techniques allow us to provide a global characterization of the system's dynamics, as well as to describe the effects of land-surface heterogeneity on flow morphology. An ongoing part of our research is the study of the statistical properties displayed by topological flow features (e.g. self-similarity).

This work is part of my PhD thesis under Prof. Angela Kunoth (Mathematical Institute, University of Cologne), and is supported by the DFG-funded research initiative SFB/TR32, 'Patterns in Soil-Vegetation-Atmosphere Systems'.

Michał Lipiński
Jagiellonian University

**Persistent homology of Morse decomposition in combinatorial dynamics.**

We investigate combinatorial dynamical systems on simplicial complexes considered as finite topological spaces. Such systems arise in a natural way from sampling dynamics and may be used to reconstruct some features of the dynamics directly from the sample. We study the homological persistence of Morse decompositions of such systems, an important descriptor of the dynamics, as a tool for validating the reconstruction. Our approach may be viewed as a step toward applying the classical persistence theory to data collected from a dynamical system.

This is a joint work with T. K. Dey, M. Juda, T. Kapela, J. Kubica and M. Mrozek.

Ugo Locatelli
University of Rome "Tor Vergata"

**A reverse KAM method to estimate unknown mutual inclinations in exoplanetary systems**

The orbital evolution of the major planets of the Solar system is usually studied in the framework of the Lagrange-Laplace secular theory (i.e, averaging the system over the fast revolution angles). In this approximation the evolution of the eccentricities and inclinations turns out to be quasi-periodic: the corresponding orbits are stable, lying on KAM tori or, at least, being close to them for times exceeding the age of the Universe. The part concerning with the orbits on invariant tori was completely proved nearly twenty years ago, for a secular model of the Sun-Jupiter-Saturn system, where the behaviour of the mutual inclination is obtained from a suitable parameter, D2, the so-called Angular Momentum Deficit.

Coming to extrasolar systems, we tackle the opposite problem: assuming that the system is stable, we aim to bound the usually unknown mutual inclination to a suitable range of values. Therefore, considering exoplanets that have been detected via Radial Velocity method, we investigate the range of values of D2 for which KAM stability applies to the secular dynamics. Such a procedure looks to be successful in providing limits on the (unknown) inclination for pairs of exoplanets having very moderate eccentricities, i.e., in situations similar to the Sun-Jupiter-Saturn system. We show our first applications to the following systems: HD141399, HD143761 and HD40307.

This work is made in joint collaboration with M. Volpi (Univ. of Namur, Belgium) and M. Sansottera (Univ. of Milan, Italy).

Sainkupar Marwein Mawiong
North-Eastern Hill University, India.

**Strong Conley Index and the properties it satisfies**

Using the strong shift equivalence of pointed space maps associated to filtration pairs for isolated invariant sets of a discrete dynamical system we define a strong Conley index independent of the choice of filtration pairs and prove that it is invariant under continuation. We show that strong Conley index satisfies the continuation property, Ważewski property. We establish that summation property of homology strong Conley index also holds.

Roy Meshulam
Technion

## Topology and Combinatorics of the Complex of Flags

Let $V$ be an $n$-dimensional space over the finite field $\mathbb{F}_q$. The complex of flags $X_V$ is the simplicial complex whose vertices are the linear subspaces $0 \neq U \subsetneq V$, and whose $k$-simplices are the chains $\{U_0 \subset \cdots \subset U_k\}$. This complex, also known as the spherical building associated to the group $GL(V)$, appears in a number of different mathematical areas, including topology, combinatorics and representation theory. We will survey some old and new results and problems around $X_V$, including:
1. Homological properties of $X_V$: the Steinberg representation and the Lusztig-Dupont local systems.
2. Coboundary expansion of $X_V$ and its applications to topological overlap theorems and property testing.
3. Extremal problems for Möbius functions of posets and lattices.

Kaori Nagato-Plum
Karlsruhe Institute of Technology

## Orbital stability investigation for travelling waves in a nonlinearly supported beam

We consider the fourth-order wave equation

$$\varphi_{tt} + \varphi_{xxxx} + f(\varphi) = 0, \ (x,t) \in \mathbb{R} \times \mathbb{R}^+,$$

with a nonlinearity $f$ vanishing at 0. Solitary traveling waves $\varphi = u(x - ct)$ satisfy the ODE

$$u'''' + c^2 u'' + f(u) = 0 \text{ on } \mathbb{R},$$

and for the case $f(u) = e^u - 1$, the existence of at least 36 travelling waves was proved in [1] by computer assisted means.

We investigate the orbital stability of these solutions via computation of their Morse indicies and using results from [2] and [3]. In order to achieve it we make use of both analytical and computer-assisted techniques.

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Michael Plum
Karlsruhe Institute of Technology

## Computer-assisted proofs for semilinear elliptic boundary value problems

For second-order semilinear elliptic boundary value problems on bounded or unbounded domains, a general computer-assisted method for proving the existence of a solution in a "close" and explicit neighborhood of an approximate solution, computed by numerical means, is proposed. To achieve such an existence and enclosure result, we apply Banach's fixed-point theorem to an equivalent problem for the error, i.e., the difference between exact and approximate solution. The verification of the conditions posed for the fixed-point argument requires various analytical and numerical techniques, for example the computation of eigenvalue bounds for the linearization at the approximate solution. The method is used to prove existence and multiplicity results for some specific examples.

Mateusz Przybylski
Jagiellonian University

## Conley index approach to sampled dynamics. Part II

We develop a method which enables us to prove the existence of some periodic orbits of $\varepsilon$-approximations, that is continuous functions contained in some neighborhood of the graph of some multivalued map. Under suitable assumptions this may be extended to prove the existence of a semiconjugacy from the dynamics of the $\varepsilon$-approximation to a shift dynamics on symbol space. The symbol space and its shift map are deduced from the components of isolating neighborhood for multivalued map and certain associated endomorphisms.

We provide algorithms constructing the shift dynamics from the time series and verifying its correctness. We illustrate the machinery on two examples. The first example concerns delayed Henon map in 3D. The second example pertains to the map generated by time series acquired as a result of measurement of magnetoelastic ribbon's behavior in varying magnetic field. Obtained results demonstrate chaotic behavior of these systems.

This talk is a continuation of the presentation by B. Batko. This is a joint work with B. Batko, K. Mischaikow and M. Mrozek.

Elena Queirolo
VU Amsterdam

### Detecting and validating bifurcations in ODEs

Validated continuation of periodic solutions of ODEs naturally brings with it the problem of bifurcation search and recognition, In particular with regards to Hopf bifurcations, that generates new periodic branches and where the singularity of the orbit causes extra difficulties.

A reformulation of ODE systems near a Hopf bifurcation is presented. Such reformulation allows standard continuation algorithms to follow the periodic orbit branch up to (and including) the Hopf bifurcation. Furthermore, it is proven that the Hopf bifurcation in the original system corresponds to a saddle node in such reformulated system. Thus, a method is propose to validate saddle node bifurcations in the context of continuation of periodic orbits. If this method is applied in conjunction with the reformulated Hopf bifurcation, then the Hopf bifurcation itself is validated. In addition, the constructed saddle node validation procedure is self standing and can be applied on its own right to any periodic solution branch of ODE during the validated continuation procedure.

Philippe Robutel
IMCCE, Observatoire de Paris

### Co-orbital motions in the planetary three-body problem: from astronomical observation to KAM theory

After a short review of the co-orbital bodies present in the solar system, an analytical Hamiltonian formalism adapted to the study of the motion of two planets or satellites in co-orbital resonance will be presented. Then, focusing on the fixed points of this approximation, some remarkable orbits will be highlighted. In the last part of the talk, I shall show how KAM theory can be applied to demonstrate the existence of "horseshoe" quasiperiodic orbits modeling the motion of Janus and Epimetheus around Saturn.

Primoz Skraba
Jozef Stefan Institute

### Persistent Structures and Stability

This talk will cover how some structures appear in random complexes particularly with respect to persistent homology. I will first cover what is known about persistent non-trivial cycles (i.e. persistent homology classes), including understanding the behaviour of relatively short bars as well as longer bars. The concentration will be primarily on geometric random complexes. The second part of the talk will be on minimum spanning acycles (also called generalized trees) and their connection to persistent homology, naturally leading to discussion of new stability results for persistence diagrams. Time permitting, I will conclude with open questions.

Kelly Spendlove
Rutgers University

### A Computational Framework for Connection Matrices

Algebraic topology and dynamical systems are intimately related: the algebra may constrain or force the existence of certain dynamics. Morse homology is the prototypical theory grounded in this observation. Conley theory is a far-reaching topological generalization of Morse theory and a great deal of effort over the last few decades has established a computational version of the Conley theory. The computational Conley theory is a blend of combinatorics, order theory and algebraic topology and has proven effective in tackling problems within dynamical systems.

Within the Conley theory the connection matrix is the mathematical object which transforms the approach into a truly homological theory; it is the Conley-theoretic generalization of the Morse boundary operator. We'll discuss how the connection matrix can be computed efficiently with discrete Morse theoretic techniques. We will also discuss a software package for such computations. Time permitting, we'll demonstrate our techniques with an application of our theory and software for computing on high-dimensional cubical complexes within the setting of a Morse theory on spaces of braid diagrams introduced by R. van der Vorst et al. This application allows us to prove forcing theorems for stationary and periodic solutions and connecting orbits in a class of parabolic PDEs.

Daniel Strzelecki
Nicolaus Copernicus University in Toruń

**Equivariant Conley index of an orbit**

The results below are described in the papers [1, 2].

The talk is devoted to computation of an equivariant Conley index $\mathcal{CI}_G(G(x_0), -\nabla\varphi)$ of an orbit $G(x_0)$ and to distinguishing such objects, where $G$ is a compact Lie group. Recall that $\wedge_{G_{x_0}}$ denotes a smash product over group defined by tom Dieck. We are going to prove the following theorem which reduce the computation to the study of Conley index of an isolated point.

**Theorem 1.** Let $\varphi \in C^2_G(\mathbb{R}^n, \mathbb{R})$ i.e. $\varphi$ is a $G$-invariant potential of the class $C^2$. Suppose that $G(x_0)$ is an isolated orbit of critical points of $\varphi$. Let $\phi \in C^2_{G_{x_0}}(\mathbb{V}, \mathbb{R})$ be the restriction of $\varphi$ to the space $\mathbb{V} = T^{\perp}_{x_0} G(x_0)$. Then

$$\mathcal{CI}_G(G(x_0), -\nabla\varphi) = G^+ \wedge_{G_{x_0}} \mathcal{CI}_{G_{x_0}}(\{x_0\}, -\nabla\phi).$$

Under some assumptions on the isotropy group $G_{x_0}$ we are able to distinguish two equivariant Conley indexes of orbits by the study of equivariant Euler characteriscits $\chi_H$ of the Conley indexes in restricted space.

**Definition** Fix $H \in \overline{\text{sub}}(G)$. A pair $(G, H)$ is said to be *admissible* if for any $K_1, K_2 \in \overline{\text{sub}}(H)$ the following condition is satisfied: if $(K_1)_H \neq (K_2)_H$ then $(K_1)_G \neq (K_2)_G$.

**Theorem 2.** Let $G(x_0'), G(x_0'') \subset (\nabla\varphi)^{-1}(0)$ be orbits of critical points of the potential $\varphi \in C^2_G(\Omega, \mathbb{R})$ such that $G_{x_0'} = G_{x_0''} (= H)$. If the pair $(G, H)$ is admissible and $\chi_H(\mathcal{CI}_H(\{x_0'\}, -\nabla\phi')) \neq \chi_H(\mathcal{CI}_H(\{x_0''\}, -\nabla\phi''))$ then

$$\mathcal{CI}_G(G(x_0'), -\nabla\varphi) \neq \mathcal{CI}_G(G(x_0''), -\nabla\varphi).$$

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Robert Szczelina
Jagiellonian University

**Some developments in rigorous forward in time integration of Delay Differential Equations**

In this talk I will describe some issues that arise in the implementation of rigorous integration scheme for Delay Differential Equations using piecewise Taylor representation of the phase space. I will also present some developments to the algorithms that produce better estimates, which should allow to extend the applicability of the method to obtain computer assisted proofs of wider spectrum of problems.

Joint work with Piotr Zgliczyński.

Hiroshi Takeuchi
Tohoku University

### The Persistent Homology of a Sampled Map: From a Viewpoint of Quiver Representations

Let $X$ and $Y$ be topological spaces, and $f: X \to Y$ be a continuous map. If we know only $X$, $Y$, and sampling data $f\restriction_S$ which is a restriction of $f$ on a finite subset $S \subset X$, then can we retrieve any information about the homology induced map $f_*: HX \to HY$?

In the paper [S. Harker, H. Kokubu, K. Mischaikow and P. Pilarczyk (2016)], they set a correspondence $F \subset X \times Y$ approximating the map $f$, and define its induced map $F_*$, which can retrieve $f_*$ under certain condition. In this study, we redefine induced maps of correspondences within the framework of quiver representations. Our definition does not need the two assumptions mentioned in the original paper, and when the two assumptions are satisfied, our definition coincides with the original definition. We reproved main theorems in the original paper only by overseeing indecomposable representations on commutative ladders.

The key idea is to decompose and focus on the interval representation $\mathbb{I}[1,3]$. This viewpoint provides another analysis for discrete dynamical systems by using eigenspace functors which is shown in the paper [H. Edelsbrunner, G. Jabłoński and M. Mrozek (2015)]. The definition is justified by algebraic property of matrix method of commutative ladders. This analysis has stability, accordingly, even if input data include noise, the obtained persistence modules and persistence diagrams can be affected to the extent of the noise.

Jan Bouwe van den Berg
VU Amsterdam

### Computer-assisted theorems for the Ohta-Kawasaki problem

In this talk we discuss a rigorous numerical method to study pattern formation in the Ohta-Kawasaki model in both two and three dimensions. This model appears in the study of di-block copolymers. The model describes, roughly speaking, the balance between long range attraction and short range repulsion.

For the two dimensional case we compare local minimizers of the Ohta-Kawasaki functional. In particular, we validate part of the phase diagram, identifying regions of parameter space where rolls are favorable, where hexagonally packed spots have lowest energy and finally where the constant mixed state does. This includes determining optimal domain sizes, which are unknown a priori. In terms of the practical realities of applying the computer assisted theorems ideas, this work represents a step forward past clean-cut test problems to more elaborate variational problems in pattern formation.

For the three dimensional case we illustrate the work in progress by fixing a cubical domain and finding several highly symmetric periodic stationary states. This is based on joint work with JF Williams.

Lucile Vandembroucq
University of Minho, Portugal

### On the topological complexity of surfaces

In this talk, we will consider the concept of topological complexity introduced by M. Farber and discuss it for surfaces. In particular, we will see that the topological complexity of the Klein bottle and higher genus non-orientable surfaces is maximal. The talk is based on a joint work with D. Cohen, which completes the results of A. Dranishnikov on the topological complexity of non-orientable surfaces of genus greater than or equal to 4.

Irmina Walawska
Jagiellonian University

### Continuation and bifurcations of Halo orbits - computer-assisted proof

We propose an algorithm for rigorous validation that a family of periodic orbits preserving some symmetries undergoes various types of bifurcations, including pe- riod doubling, tripling and quadrluping. We also give an algorithm for rigorous continuation of these orbits. The method uses rigorous computation of higher order derivatives of well chosen Poincaré map with symmetry properties of the system. As an application we give a computer assisted proof that the Halo orbits bifurcate from the family of Lyapunov orbits for wide range of the parameters $\mu$. For $\mu$ corresponding to the Sun-Jupiter and Earth-Moon system we give a proof of the existence of a wide continuous branch of Halo orbits that undergo period $k$-tupling bifurcations. The computer assisted proof uses rigorous ODE solvers and algorithms for computation of Poincare maps from the CAPD library.

This is a joint work with D. Wilczak.

Krzysztof Ziemiański
Polish Academy of Sciences

**Stable components of directed spaces**

Directed spaces are objects that can be used for modeling behavior of concurrent programs. A directed space is a topological space, whose points represent possible states of a program, with a family of distinguished paths, which represent possible executions. Directed spaces allow for analysis of concurrent programs with means of algebraic topology.

On of the problems of directed algebraic topology is that many classical, non-directed homotopy invariants of topological spaces have no satisfactory directed counterparts. In my talk, I will consider decompositions of directed spaces into a finite number of disjoint subsets that satisfy some "stability" axioms. I will show that a large class of directed spaces admits the "coarsest" decomposition into stable components. Next, I will define the component category for a system of stable components; this category is enriched in the homotopy category. I hope that such defined component categories will play a part of homotopy groups in directed algebraic topology.