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A hypothetical upper bound for solutions of a Diophantine equation with a finite number of solutions

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Abstract. Let $E_n = \{x_i = 1,\ x_i + x_j = x_k,\ x_i \cdot x_j = x_k :\ i,j,k \in \{1,...,n\}\}$, $\boldsymbol{K} \in \{\mathbb{Z},\ \mathbb{Q},\ \mathbb{R},\ \mathbb{C}\}$. We construct a system $S \subseteq E_{21}$ such that S has infinitely many integer solutions and S has no integer solution in $[-2^{2^{21-1}}, 2^{2^{21-1}}]^{21}$. We conjecture that if a system $S \subseteq E_n$ has a finite number of solutions in $\boldsymbol{K}$, then each such solution $(x_1,...,x_n)$ satisfies $(|x_1|,...,|x_n|) \in [0, 2^{2^{n-1}}]^n$. Applying this conjecture for $\boldsymbol{K} = \mathbb{Z}$, we prove that if a Diophantine equation has only finitely many integer (rational) solutions, then the heights of solutions are bounded from above by a constant which recursively depends on the coefficients of the equation. We note that an affirmative answer to the famous open problem whether each listable set $\mathcal{M} \subseteq \mathbb{Z}^n$ has a finite-fold Diophantine representation would falsify our conjecture for $\boldsymbol{K} = \mathbb{Z}$.