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Cordial labeling of hypertrees

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Abstract. Let $H = (V, E)$ be a hypergraph with vertex set $V = \{v_1, \ldots, v_n\}$ and edge set $E = \{e_1, \ldots, e_m\}$. A vertex labeling $c : V \to \mathbb{N}$ induces an edge labeling $c^* : E \to \mathbb{N}$ by the rule $c^*(e_i) = \sum_{v_j \in e_i} c(v_j)$. For integers $k \geq 2$ we study the existence of labelings satisfying the following condition: Every residue class modulo k occurs exactly $\lfloor n/k \rfloor$ or $\lceil n/k \rceil$ times in the sequence $c(v_1), \ldots, c(v_n)$ and exactly $\lfloor m/k \rfloor$ or $\lceil m/k \rceil$ times in the sequence $c^*(e_1), \ldots, c^*(e_m)$. Hypergraph H is called k-cordial if it admits a labeling with these properties.

Hovey [Discrete Math. 93 (1991), 183–194] raised the conjecture (still open for $k > 5$) that if H is a tree graph, then it is k-cordial for every k. Here we investigate the analogous problem for hypertrees (connected hypergraphs without cycles) and present various sufficient conditions on H to be k-cordial. From our theorems it follows that every k-uniform hypertree is k-cordial, and every hypertree with n or m odd is 2-cordial. Both of these results generalize Cahit's theorem [Ars Combin. 23 (1987), 201–207] which states that every tree graph is 2-cordial. We also prove that every uniform hyperpath is k-cordial for every k.

Key words. k-cordial graph, hypergraph, hypergraph labeling labeling, hypertree